College of the Holy Cross, Spring 2014
Math 136, section 1 – Midterm Exam 2
Friday, March 28

Your Name: ______________________________

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

Problem	Points/Poss
I	/ 30
II	/ 15
III	/ 15
IV	/ 30
V	/ 10
Total	/100

I. For these problems, you must show *all work necessary* to justify your answers, but you may consult the portions of the table of integrals provided as needed. If you use a table entry, identify it by number.

A. (15) Integrate with the partial fraction method: $\displaystyle\int \frac{3x+1}{x^3+16x}\,dx$

Answer:

B. (15) Integrate via a trigonometric substitution: $\int x^3\sqrt{81 - x^2}\ dx$

Answer:

II.

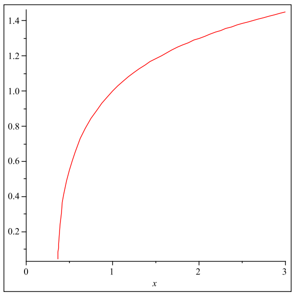
A. (5) Use midpoint Riemann sums with $n = 2$ to approximate $\int_1^2 \sqrt{1 + \ln(x)}\, dx$.

Midpoint approximation:

B. (5) Use the trapezoidal rule with $n = 2$ to approximate $\int_1^2 \sqrt{1 + \ln(x)}\, dx$.

Trapezoidal rule approximation:

C. (5) The function $y = \sqrt{1 + \ln(x)}$ is plotted below.



Check the appropriate boxes:

The midpoint approximation is a overestimate □/underestimate □.

The trapezoidal rule approximation is a overestimate □/underestimate □.

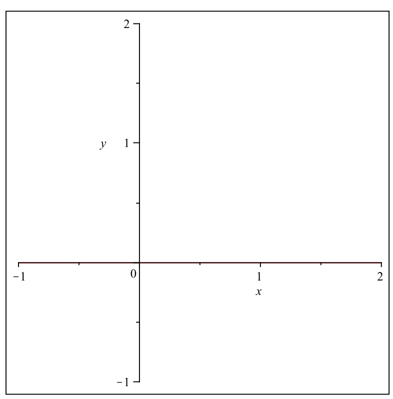
III. For each of the following improper integrals, set up and evaluate the appropriate limits to determine whether the integral converges. If so, find its value; if not, say "does not converge." (Credit will be given only for the correct limit calculation.)

A. (5) $\int_1^\infty e^{-x}\, dx$

B. (10) $\int_{-1}^{1} \frac{1}{x^2}\, dx$

IV. The region R is bounded by the graphs $y = \sqrt{x}$ and $y = x^2$.

A. (5) Sketch the region R.



B. (5) Set up and compute an integral to find the area of R.

Area = [ ]

C. (10) The region R is rotated about the x-axis to generate a solid. Set up and compute an integral to find its volume.

Volume = [ ]

D. (10) A solid has the region R as base and cross-sections by planes perpendicular to the x-axis are isosceles right triangles with hypotenuse extending from the lower boundary to the upper boundary of the region. Set up and compute an integral to find the volume.

Volume = [ ]

V. (10) Suppose that the region R defined by $0 \leq y \leq f(x)$ and $a \leq x \leq b$ has area A and lies above the x-axis. When R is rotated about the x-axis it sweeps out a solid with volume V_1. When R is rotated about the line $y = -k$, where $k > 0$, it sweeps out a solid with volume V_2. Express V_2 in terms of V_1, k, A.

Volume $V_2 =$ [ ]