College of the Holy Cross, Spring 2014<br>Math 136, section 1 - Makeup Midterm Exam 2<br>Monday, March 31

Your Name: $\qquad$

Instructions: For full credit, you must show all work on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

| Problem | Points/Poss |
| :--- | ---: |
| I | $/ 30$ |
| II | $/ 15$ |
| III | $/ 15$ |
| IV | $/ 30$ |
| V | $/ 10$ |
| Total | $/ 100$ |

I. For these problems, you must show all work necessary to justify your answers, but you may consult the portions of the table of integrals provided as needed. If you use a table entry, identify it by number.
A. (15) Integrate with the partial fraction method: $\int \frac{3 x+1}{x^{3}+6 x^{2}+5 x} d x$

Answer:
B. (15) Integrate via a trigonometric substitution: $\int \frac{x^{2}}{\sqrt{4+x^{2}}} d x$

Answer:
II.
A. (5) Use a midpoint Riemann sum with $n=2$ to approximate $\int_{1}^{2} \sqrt{1+\ln (x)} d x$.

Midpoint approximation: $\square$
B. (5) Use the trapezoidal rule with $n=2$ to approximate $\int_{1}^{2} \sqrt{1+\ln (x)} d x$.
$\square$
C. (5) The function $y=\sqrt{1+\ln (x)}$ is plotted below.


Check the appropriate boxes:
The midpoint approximation is a overestimate $\square /$ underestimate $\square$.
The trapezoidal rule approximation is a overestimate $\square /$ underestimate $\square$.
III. For each of the following improper integrals, set up and evaluate the appropriate limits to determine whether the integral converges. If so, find its value; if not, say "does not converge." (Credit will be given only for the correct limit calculation.)
A. (5) $\int_{1}^{\infty} \frac{1}{1+x^{2}} d x$
B. (10) $\int_{-1}^{1} \frac{1}{x} d x$
IV. The region $R$ is bounded by the graphs $y=\sqrt{x}$ and $y=x^{3}$.
A. (5) Sketch the region $R$.

B. (5) Set up and compute an integral to find the area of $R$.

$$
\text { Area }=\square
$$

C. (10) The region $R$ is rotated about the $x$-axis to generate a solid. Set up and compute an integral to find its volume.

$$
\text { Volume }=\square
$$

D. (10) A solid has the region $R$ as base and cross-sections by planes perpendicular to the $x$-axis are equilateral triangles with side extending from the lower boundary to the upper boundary of the region. Set up and compute an integral to find the volume.

$$
\text { Volume }=
$$


V. (10) Suppose that the region $R$ defined by $0 \leq y \leq f(x)$ and $a \leq x \leq b$ has area $A$ and
lies above the $x$-axis. When $R$ is rotated about the $x$-axis it sweeps out a solid with volume $V_{1}$. When $R$ is rotated about the line $y=-k$, where $k>0$, it sweeps out a solid with volume $V_{2}$. Express $V_{2}$ in terms of $V_{1}, k, A$.

Volume $V_{2}=$

