College of the Holy Cross, Spring Semester, 2014
MATH 136, Section 01, Final Exam
Wednesday, May 14, 11:30 AM
Professor Little

Your Name: ______________________________

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values. There are 200 regular points and 10 possible Extra Credit points (see problem IV).

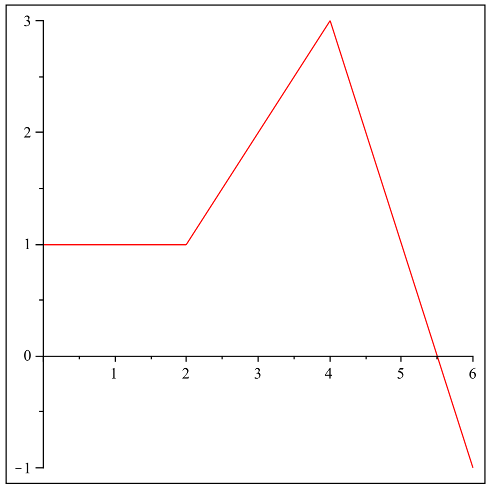
Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 50
III	/ 20
IV	/ 40
V	/ 20
VI	/ 25
VII	/ 25
Total	/200

Have a safe, productive, and enjoyable summer!

I.

(A) (10) Let $f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 2 \\ x-1 & \text{if } 2 < x \le 4 \\ 11-2x & \text{if } 4 < x \le 6 \end{cases}$. whose graph is shown here:



Let $F(x) = \int_0^x f(t)\,dt$. Complete the following table of values for $F(x)$:

x	0	1	2	3	4	5	6
$F(x)$	0						

(B) (10) Compute the derivative of the function $g(x) = \displaystyle\int_0^{2x} \frac{\cos(t)}{t^2}\,dt$.

$g'(x) =$

II. Compute the following integrals. Some of these may be forms covered by entries in the table of integrals. If you use a table entry, state which one.

(A) (5) $\displaystyle\int \frac{x^4 - 3\pi^3 + \sqrt{x}}{x^{2/3}}\,dx$

Answer

(B) (5) $\displaystyle\int xe^{x^2}\,dx$

Answer

(C) (10) $\displaystyle\int \frac{\csc^2(5x)\,dx}{\cot(5x)+7}$

Answer

(D) (10) $\displaystyle\int_1^e x^5 \ln(x)\,dx.$

Answer

(E) (10) $\displaystyle\int \frac{1}{\sqrt{16+x^2}}\,dx$

Answer

(F) (10) $\int \frac{1}{x^3+x^2+x+1}\,dx$

Answer

III.

(A) (5) Set up an integral to compute the length of the curve $y = x^3$ from $x = 1$ to $x = 3$.

Answer $\boxed{}$

(B) (10) Use a midpoint Riemann sum with $n = 4$ to approximate your integral from part (A).

Answer $\boxed{}$

(C) (5) Given: The graph of the function in the arclength integral is concave up on the interval $[1, 3]$. Check the appropriate box:

The midpoint approximation is a overestimate □/underestimate □.

IV. A region R in the plane is bounded by the graphs $y = 9 - x^2$, $y = 2x$, $x = 0$ and $x = 1$.

(A) (20) Compute the area of the region R.

Area = $\boxed{}$

(B) (20) Compute the volume of the solid obtained by rotating R about the x-axis.

Volume = $\boxed{}$

(C) *Extra Credit (10)* Set up integral(s) to compute the volume of the solid obtained by rotating R about the y-axis. *You do not need to compute the value.*

Volume Integral(s) = $\boxed{}$

V. (20) The daily solar radiation x per square meter (in hundreds of calories) in Florida in October has a probability density function $f(x) = c(x-2)(6-x)$ if $2 \le x \le 6$, and zero otherwise. Find the value of c, and then compute the probability that the daily solar radiation is > 4 hundred calories per square meter.

$c =$ $\boxed{}$

Probability = $\boxed{}$

VI. An avian flu epidemic has broken out in Birdsburgh, a large city with total population 10 million. Let N be the number, in millions, of people who have been infected, as a function of time t in weeks. The Birdsburgh Public Health department determines that the rate of change of N is proportional to the product of the number of infected people (N) and the number of people not yet infected.

(A) (10) Write the statement above as a differential equation, calling the constant of proportionality k.

Differential Equation:

(B) (5) The function $N(t) = 10/(1 + 9999e^{-t})$ should be a solution of your differential equation from part A. What is the value of k?

$k =$

C) (10) If the epidemic proceeds according the function given in part B, how many weeks will pass before the number of infected people reaches 1 million?

Time to reach 1 million = $\boxed{}$

VII. (25) Using the Ratio Test and testing the endpoints, determine the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{4^n x^n}{n^2}$. Explain.

Interval of convergence $\boxed{}$