1. Compute the indicated limits. Show all work for full credit.

   (a) (5) $\lim_{x\to 1} \frac{5x^2 - 3x - 2}{x^2 - 7x + 6}$

   Solution: The top and bottom are going to zero separately as $x \to 1$, so we need to do some algebra and try to cancel factors:

$$\begin{aligned} \lim_{x\to 1} \frac{5x^2 - 3x - 2}{x^2 - 7x + 6} &= \lim_{x\to 1} \frac{(5x+2)(x-1)}{(x-6)(x-1)} \\ &= \lim_{x\to 1} \frac{5x+2}{x-6} \\ &= -\frac{7}{5}. \end{aligned}$$

   (b) (5) $\lim_{x\to 2} \frac{5x^2 - 3x - 2}{x^2 - 7x + 6}$

   Solution: Now, neither the numerator nor the denominator is going to 0 as $x \to 2$, so the rational function is continuous at $x = 2$ and the limit is

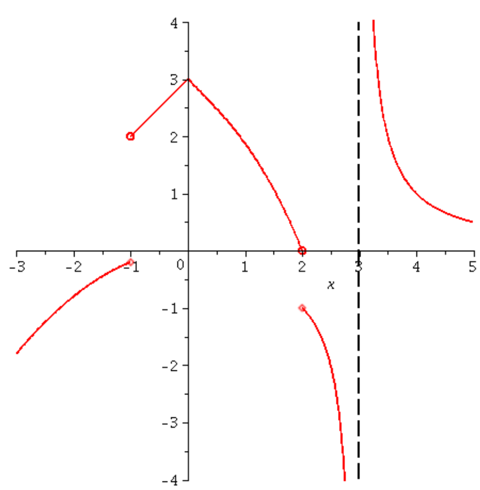
$$\lim_{x\to 2} \frac{5x^2 - 3x - 2}{x^2 - 7x + 6} = \frac{12}{-4} = -3$$

   (c) (5) $\lim_{x\to \infty} \frac{5x^2 - 3x - 2}{x^2 - 7x + 6}$

   Solution: The limit is 5, as can be seen by this calculation (multiply top and bottom by $\frac{1}{x^2}$):

$$\begin{aligned} \lim_{x\to \infty} \frac{5x^2 - 3x - 2}{x^2 - 7x + 6} &= \lim_{x\to \infty} \frac{5 - \frac{3}{x} - \frac{2}{x^2}}{1 - \frac{7}{x} + \frac{6}{x^2}} \\ &= 5. \end{aligned}$$

2. The graph of a function f with $f(-1) = -.2$ and $f(2) = -1$ is shown below.



(a) (10) What are $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$?

Solution: From the graph, $\lim_{x\to 2^-} f(x) = 0$ and $\lim_{x\to 2^+} f(x) = -1$.

(b) (15) Find all x in $(-3, 5)$ where f is discontinuous. Explain.

Solution: $f(x)$ has jump discontinuities at $x = -1$ and $x = 2$. It also has an infinite discontinuity (vertical asymptote) at $x = 3$. These are the only discontinuities.

(c) (10) Given that $f(x) = x + 3$ for $-1 < x < 0$ and $f(x) = 3 - x - \frac{x^3}{8}$ for $0 \le x < 2$, is f differentiable at $a = 0$? Why or why not?

Solution: The answer is *no*, f is not differentiable at 0. The easiest way to see this is that $\lim_{h\to 0^-} \dfrac{f(0+h) - f(0)}{h}$ will agree with the derivative of $x + 3$ at $x = 0$, and equal 1. On the other hand $\lim_{h\to 0^+} \dfrac{f(0+h) - f(0)}{h}$ will agree with the derivative of $3 - x - \frac{x^3}{8}$ at $x = 0$, which is -1. (This is the precise meaning of the apparent "corner" on the graph at $x = 0$.) Since the one-sided limits of the difference quotient of f are not the same, $f'(0)$ does not exist.

3. Use the sum, product, and/or quotient rules to compute the following derivatives. You may use any correct method, but must show work and simplify your answers for full credit.

(a) (5) $\dfrac{d}{dx}\left(\dfrac{5}{\sqrt{x}} - e^x + 3\right)$

Solution: By the sum, power and exponential rules,

$$\frac{d}{dx}\left(\frac{5}{\sqrt{x}} - e^x + 3\right) = \frac{-5}{2}x^{-3/2} - e^x$$

(b) (10) $\dfrac{d}{du}(u^{5/3}e^u)$

Solution: By the product, power and exponential rules,

$$\frac{d}{du}(u^{5/3}e^u) = u^{5/3}e^u + \frac{5}{3}u^{2/3}e^u = u^{2/3}\left(u + \frac{5}{3}\right)e^u.$$

(c) (10) $\dfrac{d}{dv}\left(\dfrac{v^3 - 2v}{v^2 + 5v + 1}\right)$

Solution: By the quotient rule

$$\begin{aligned}\frac{d}{dv}\left(\frac{v^3 - 2v}{v^2 + 5v + 1}\right) &= \frac{(v^2 + 5v + 1)(3v^2 - 2) - (v^3 - 2v)(2v + 5)}{(v^2 + 5v + 1)^2} \\ &= \frac{v^4 + 10v^3 + 5v^2 - 2}{(v^2 + 5v + 1)^2}.\end{aligned}$$

(d) (5) $\frac{d}{dx}\left(\frac{e^\pi + \pi^e - x^\pi}{4}\right)$

Solution: Since e^π and π^e are constants, the derivative is just $\frac{-\pi x^{\pi-1}}{4}$.

4. Do not use the differentiation rules from Chapter 3 in this question.

(a) (5) State the limit definition of the derivative $f'(x)$.

Solution: The derivative $f'(x)$ is

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h},$$

if the limit exists.

(b) (10) Use the definition to compute the derivative function of $f(x) = \sqrt{x+1}$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \\ &= \lim_{h\to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \\ &= \frac{1}{2\sqrt{x+1}}. \end{aligned}$$

(c) (5) Find the equation of the line tangent to the graph $y = \sqrt{x+1}$ at $x = 8$.

Solution: By the previous part, the slope of the tangent is $f'(8) = \frac{1}{6}$. The tangent line has equation:

$$y - 3 = \frac{1}{6}(x - 8)$$