

# Math/Music: Aesthetic Links

Gareth E. Roberts

Department of Mathematics and Computer Science  
College of the Holy Cross  
Worcester, MA

Béla Bartók and the Golden Section  
April 8, 2011

## Béla Bartók

- Born in Nagyszentmiklós Hungary (now Sînnicolau Mare, Romania) in 1881. Died in New York, Sept. 1945.
- Could play 40 songs on the piano by age 4. Writes first piece of music at age 6. Quickly becomes a chapel organist and an accomplished pianist and composer.
- Studies at the Catholic Gymnasium (high school) in Pozsony where he excels in **math** and physics in addition to music. Enters the Academy of Music (Liszt is 1st president) in Budapest in 1899.
- Avid collector of folk music (particularly Hungarian, Romanian and Slovakian).
- Influenced by Debussy and Ravel; preferred Bach to Beethoven.
- Considered to be one of Hungary's greatest composers (along with Franz Liszt).

## Béla Bartók (cont.)



Figure: Bartók at age 22.

- Very interested in **nature**. Builds impressive collection of plants, insects and minerals. Fond of sunflowers and fir-cones.
- “We follow nature in composition ... folk music is a phenomenon of nature. Its formations developed as spontaneously as other living natural organisms: the flowers, animals, etc.” — Bartók in *At the Sources of Folk Music* (1925)
- Notoriously silent about his own compositions. “Let my music speak for itself, I lay no claim to any explanation of my works!”

## Ernö Lendvai

- Beginning in 1955, the Hungarian musical analyst **Ernö Lendvai** starts to publish works claiming the existence of the **Fibonacci numbers** and the **golden ratio** in many of Bartók's pieces.
- Some find Lendvai's work fascinating and build from his initial ideas; others find errors in his analysis and begin to discredit him. Lendvai becomes a controversial figure in the study of Bartók's music.
- Lendvai draws connections between Bartók's love of nature and "organic" folk music, with his compositional traits. He takes a broad view, examining form (structure of pieces, where climaxes occur, phrasing, etc.) as well as tonality (modes and intervals), in discerning a substantial use of the golden ratio and the Fibonacci numbers.

## Example: *Music for Strings, Percussion and Celesta*, Movement I

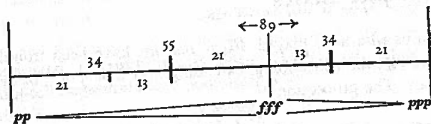


FIG. 22

Lendvai's analysis states:

- 1 Piece is 89 measures long.
- 2 The climax of the movement occurs at the end of bar 55 (loudest moment), which gives a subdivision of two Fibonacci numbers (34 and 55) that are an excellent approximation to the golden ratio.
- 3 Violin mutes begin to be removed in bar 34 and are placed back on in bar 69 ( $56 + 13 = 69$ ).
- 4 The exposition in the opening ends after 21 bars.

# Music for Strings, Percussion and Celesta

## INSTRUMENTATION

Strings to be placed in two different groups

1st Group : Violins I and II, Viola I  
Violoncello I, Double Bass I

2nd Group : Violins III and IV, Viola II  
Violoncello II, Double Bass II

Side Drum without snares  
Side Drum with snares  
\*Cymbals  
Tam-Tam  
Bass Drum

} one player

\*\*Timpani (chromatic)

Xylophone

Celesta (takes over at certain places the second part  
of the piano)

Harp

Pianoforte

## APPROXIMATE POSITION OF THE ORCHESTRA

	Double Bass I	Double Bass II	
Violoncello I	Timpani	Bass Drum	Violoncello II
Viola I	Side Drums	Cymbals	Viola II
Violin II	Celesta	Xylophone	Violin IV
Violin I .	Pianoforte	Harp	Violin III

DURATION : 26 Minutes

1st Mov. : 6 Min. 30 sec.

3rd Mov. : 6 Min. 35 sec.

2nd Mov. : 6 Min. 55 sec.

4th Mov. : 5 Min. 40 sec.

This work, completed in September 1936, was first performed at Bale on  
21st January 1937, conducted by Paul Sacher.

## GENERAL REMARKS

(1) All glissandi marked *السر* etc. in the string, as well as in the timpani parts, are to be played in such a manner that the starting note is left immediately, so that an even gliding sound during the full value of the first note is produced.

(2) The 4th movement can, in special circumstances, be played a little slower for acoustic reasons.

\* 2 pairs, one of which should be of smaller size (sounding higher).

\*\* If no chromatic Timpani available, the Timpani glissandi should be played on the piano in corresponding chromatic passages. For this purpose an extra player, not instrument, is required.

Musik für Saiteninstrumente,  
Schlagzeug und Celesta (in 4 Sätzen)

1

Musique pour instruments à cordes,  
percussion et célesta (en 4 parties)

I.

Andante tranquillo,  $\text{♩}$  ca. 116-112

Béla Bartók

1. 2. Violo *con sord.* *pp* [5]

1. 4. VI. *con sord.* *pp*

2. VI. *con sord.* *pp*

1. 4. VI. *con sord.* *pp*

2. VI. *con sord.* *pp*

2. VI. *con sord.* *pp*

1. VI. [10] *con sord.* *pp*

1. 4. VI. *con sord.* *pp*

2. VI. *con sord.* *pp*

2. VI. *con sord.* *pp*

2. VI. [15]

1. 4. VI. *con sord.* *pp*

2. VI. *con sord.* *pp*

2. VI. *con sord.* *pp*

Copyright 1937 by Universal Edition.  
Copyright assigned 1939 to Boosey & Hawkes, Ltd.,  
for the British Empire, North, Central and South America.

B. & H. 16155

2. VI.  
3.4. VI.  
1.2. Vle.  
1.2. Vlo.  
1.2. Cb.

oon sord  
g pp

7 8 12

20

2. VI.  
3.4. VI.  
1.2. Vle.  
1.2. Vlo.  
1.2. Cb.

8 7 10

2. VI.  
3.4. VI.  
1.2. Vle.  
1.2. Vlo.  
1.2. Cb.



25

con sord.

3

30

Imp.

pp

senza sord.

senza sord.

senza sord.

senza sord.



35

Timp. *mf*  
 senza sord.  
 1.VI. *p*  
 2.VI. *p*  
 3.4.VI. *p*  
 1.2.Vle. *p*  
 1.2.Vlc. *p*  
 1.2.Cb. *p*  
*mf* *cresc.*  
*mf* *cresc.*

ca 120 - 126

40

Timp.  
 2.VI. *mf* *cresc.*  
 3.4.VI. *mf* *cresc.*  
 1.2.Vle. *mf* *cresc.*  
 1.2.Vlc. *mf* *cresc.*  
 1.2.Cb. *mf* *cresc.*

2.VI.  
 3.4.VI.  
 1.2.Vle.  
 1.2.Vlc.  
 1.2.Cb.



45

1.2.Vi. *sempre cresc.*

2.Vi. *sempre cresc.*

3.4.Vi. *sempre cresc.*

1.2.Vle. *f sempre cresc.*

1.2.Vlc. *sempre cresc.*

1.2.Cb. *sempre cresc.*

50

1.2.Vi. *ca. 120 - 116*

3.4.Vi. *pp*

1.2.Vle. *pp*

1.2.Vlc. *pp*

1.2.Cb. *pp*

Piatti *pp*

Timp. *(non div.)*

1.2.Vi. *(non div.)* *cresc.*

3.4.Vi. *(non div.)* *cresc.*

1.2.Vle. *(non div.)* *cresc.*

1.2.Vlc. *(non div.)* *cresc.*

1.2.Cb. *ff* *cresc.*

55

Cr. Dr.

Timp.

(non div.)

1. VI.

(non div.)

2. VI.

(non div.)

3.4. VI.

(non div.)

1.2. Vle.

(non div.)

1.2. Vlc.

1.2. Cb.

60

(non div.)

1.2. VI.

3.4. VI.

(non div.)

1.2. Vle.

1.2. Vlc.

1.2. Cb.

poco rall.

1. VI.

2. VI.

3.4. VI.

1.2. Vle.

1.2. Vlc.

1.2. Cb.

*mf*

*p*

*a*

65

tempo ca. 116 - 112

7

1. VI.  
2. VI.  
3. & 4. VI.  
1. & 2. Vio.  
1. & 2. Vio.  
1. & 2. Cb.

con sord.  
(p)

70

3. & 4. VI.  
1. & 2. Vio.  
1. & 2. Vio.

con sord.  
(p) II.  
con sord.  
(p)

2. VI.  
3. VI.  
4. VI.  
1. Vio.  
2. Vio.  
1. & 2. Vio.

con sord.  
piu p  
piu p  
piu p  
piu p  
piu p



75

2. VI.  
3. VI.  
4. VI.  
1. Vle.  
2. Vle.  
1. & 2. Vle.

ca. 108

Cel.

con sord.

pp

1. VI.  
2. VI.  
3. VI.  
4. VI.  
1. Vle.  
2. Vle.  
1. & 2. Vle.

Musical score for measures 10-80. The score includes parts for Cello (Cel.), Violins (1.VI., 2.VI., 3.VI., 4.VI.), and Violas (1.VIe., 2.VIe., 1.2.VIe.). The Cello part features a complex rhythmic pattern with sixteenth and thirty-second notes. The string parts are primarily sustained chords with some melodic movement. The key signature has one flat (B-flat), and the time signature is 8/8.

Musical score for measures 80-110. The score includes parts for Cello (Cel.), Violins (1.VI., 2.VI., 3.VI., 4.VI.), Violas (1.VIe., 2.VIe., 1.2.VIe.), and Double Bass (1.2.Cb.). A box containing the number '80' is positioned above the Cello staff. The Cello part continues with its complex rhythmic pattern. The string parts remain largely sustained. The key signature has one flat, and the time signature is 8/8. The score concludes with a *pp* (pianissimo) dynamic marking.





## Problems with Lendvai's Analysis (Roy Howat)

- 1 The piece is 88 bars long, not 89! Lendvai includes a **footnote**: “The 88 bars of the score must be completed by a whole-bar rest, in accordance with the Bülow analyses of Beethoven.” **Hanh?!**
- 2 The dynamic climax of the piece is certainly at the end of bar 55. But the tonal climax is really at bar 44, when the subject returns a tritone away from the opening A to E $\flat$ . ( $88/2 = 44$ , symmetry?)
- 3 The viola mutes come off at the end of bar 33 (not 34). The violin mutes are placed back on at the end of bar 68 (not 69). This last fact actually helps the analysis since  $68 = 55 + 13$ , giving the second part of the movement a division of 13 and 20 (21 if you allow the full measure rest at the end).
- 4 The fugal exposition actually ends in bar 20, not 21.
- 5 What about the celesta? It enters after bar 77.  $77 - 55 = 22$  (close to Fibonacci). Lendvai neglects to mention this key feature.

Fig. 4: Fugue from *Music for Strings, Percussion and Celeste*

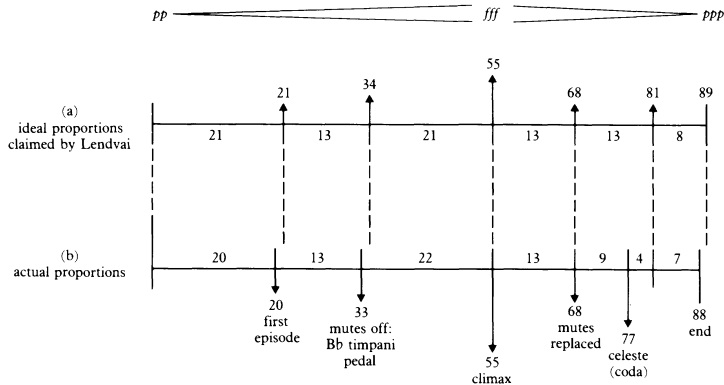


Figure: Roy Howat's analysis of Lendvai's work, from "Bartók, Lendvai and the Principles of Proportional Analysis," *Music Analysis*, 2, No. 1 (March, 1983), pp. 69-95.

## Key Features of Movement I

- The piece is a fugue (think Bach) with the opening **subject** played by the viola starting on A. This chromatic theme only ranges a tritone in distance (6 halfsteps) to E $\flat$ .
- The successive entrances of the main theme alternate between ascending perfect 5ths (A  $\rightarrow$  E  $\rightarrow$  B  $\rightarrow$  F $\sharp$  etc.) and descending perfect 5ths (A  $\rightarrow$  D  $\rightarrow$  G  $\rightarrow$  C etc.). This serves to keep each subject musically “close,” as demonstrated by the circle of fifths. The themes come back to the same note a tritone away, on E $\flat$  at the **tonal** climax of the movement (bar 44), exactly half way through the piece.
- In the second half of the piece (after the dynamic climax), the subject is inverted (exactly) and moves back around the circle of fifths to return to the opening A. Often, only the opening 5 notes are used (e.g., measure 65). The original 5-note opening of the subject returns in measure 82, dividing the **coda** (defined by the entrance of the celesta) into 4 + 7 bars.

## Key Features of Movement I (cont.)

- The opening four bars (where the main theme is announced) are subdivided into 3's and 2's. For example, 3 + 3 + 2 in measure 1 and 3 + 3 + 3 + 3 in measure 2.
- The first **stretto** in the Fugue (where the initial subject is interrupted by another entrance of the subject before completing), occurs just before the end of measure 26 on the pitches F $\sharp$  (1st) and then C (2nd). These are precisely 1/2 way around the circle of fifths. These return in inversion in the second half of the movement, ending in measure 68, giving a golden section split of 42 : 26.
- A very noticeable exact inversion of the theme occurs over the last three measures between the first and second violins. This inversion is about A, reaffirming it as the tonal center of the movement. The inversion rises and falls a tritone to E $\flat$  in both voices, mimicking the overall harmonic structure of the piece.

Fig. 5: Fugue from Music for Strings, Percussion and Celeste

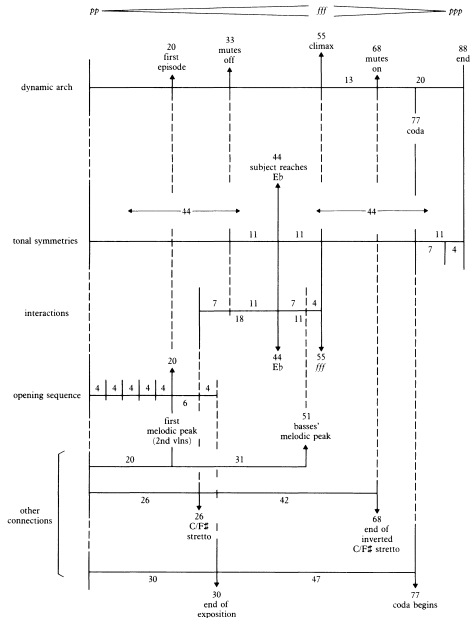
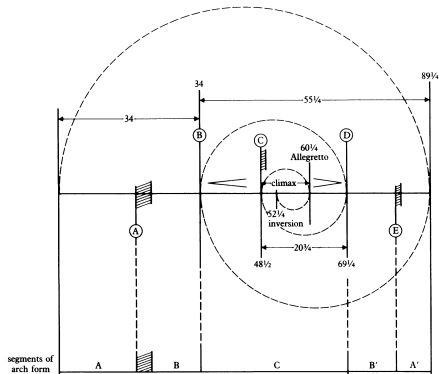


Fig 7: third movement of Music for Strings, Percussion and Celeste



The opening xylophone solo in the third movement has the rhythmic pattern of

1, 1, 2, 3, 5, 8, 5, 3, 2, 1, 1

with a crescendo followed by a decrescendo ([hairpin](#)) climaxing at the top of the sequence. Obvious nod to Fibonacci.

Ex. 2: Facsimile of recto pages 1 and 2 from manuscript 80FSS1 in the New York Béla Bartók Archive, reproduced by kind permission of Dr Benjamin Suchoff, Trustee of the Bartók Estate.

$4 = 3 + 4 - 4 + 3$

11  
7  
18  
11

---

11  
7  
18  
11  
7  
7  
18  
11

$\frac{54 \times 3}{162}$

♩ ♪ (♩)

$F = 463$

$\frac{11}{16}$   $\frac{4}{16}$   $\frac{7}{16}$   $\frac{11}{16}$   $\frac{7}{16}$   $\frac{11}{16}$

Be-yi de-ji con-de-ya Uyma-da-ya de-ya tu-ma-ya de-ya tu-ma-ya

$\text{♩} = 83$  : kemence-pöneh; str.: Ripete Sadik (16.70), 1930.  
Vilayet Rize! Artvin

**Figure:** If you dig deep enough ... Bartók's analysis of a Turkish folk song showing the Lucas numbers.

## Other Composers' Influence on Bartók

- Zoltán Kodály (1882-1967): Hungarian composer, collector of folk music, interested in music education (“Kodály Method”).
- Kodály befriends Bartók around 1905-1906. They bond over their mutual interest in folk music (Kodály was collecting phonograph cylinders of folk music in the remote areas of Hungary).
- In 1907, Kodály writes *Méditation sur un motif de Claude Debussy*. Just as with the fugue from Bartók’s *Music for Strings, Percussion and Celesta*, the piece opens *pp* and ends *ppp*, with a central climax marked *fff*. If one counts quarter notes rather than measures, there are 508 beats. The golden ration of 508 is 314 (to the nearest integer) and this just happens to be smack in the middle of the two climatic bars at *fff*.



## Claude Debussy (1862-1918)

- As Kodály was bringing Debussy to Bartók's attention, Debussy composes some interesting piano pieces whose form demonstrates the golden ratio.
- *Images*, published in 1905, consists of three piano pieces: *Reflets dans l'eau*, *Mouvement* and *Hommage à Rameau*. These soon became part of Bartók's piano repertoire.
- *Reflets* and *Mouvement* begin *pp* and finish *ppp* or *pp*, respectively. They also have main climaxes at *ff* and *fff*, respectively, located at places that divide the total piece into two portions in the golden ratio.
- *Hommage à Rameau* has a similar structure dynamically and, according to Roy Howat's analysis, "is built very clearly on Fibonacci numbers."

## Reflets dans l'eau, Debussy

Fig. 3.1

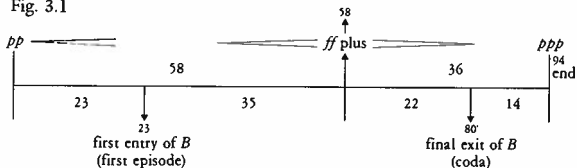
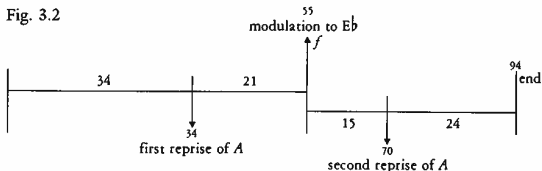


Fig. 3.2



Analysis given by [Roy Howat](#) in *Debussy in proportion: A musical analysis*, Cambridge University Press, 1983.

# Reflets dans l'eau, Tonal Structure

Fig. 3.3

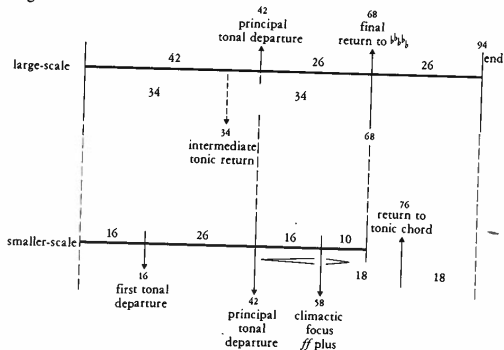
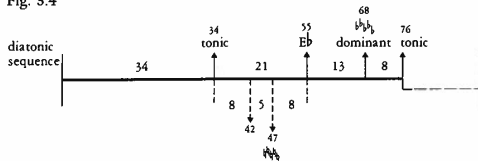
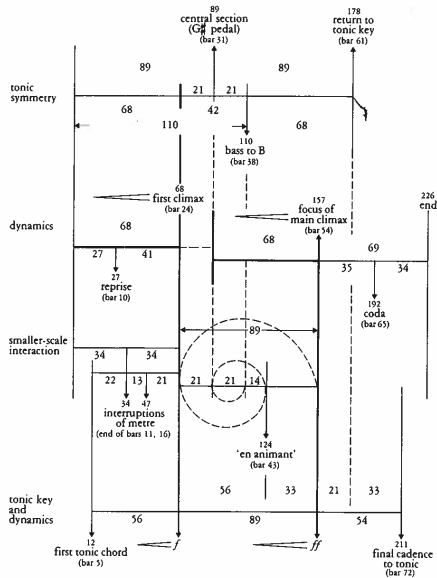


Fig. 3.4



# Hommage à Rameau, Debussy

Fig. 10.4: 'Hommage à Rameau'



## Final Remarks

- Lendvai's inaccuracies partly due to a narrow focus on the Fibonacci numbers. It's clear that the **Lucas numbers** were more significant in the first movement of *Music for Strings, Percussion and Celesta*.
- Other works by Bartók where the golden ratio can be detected are *Sonata for Two Pianos and Percussion*, *Miraculous Mandarin*, and *Divertimento*.
- Bartók was highly secretive about his works. Surviving manuscripts of many of the pieces where the golden ratio appears to have been used contain no mention of it.
- Bartók was already being criticized for being too “cerebral” in his music. Identifying the mathematical patterns in structure and tonality (even to his students!) would have only added fuel to the fire.