Seminar in Mathematics and Climate
Homework Assignment #3
Due Thurs., Feb. 22, start of class

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are encouraged to work on these problems with other classmates, and it is ok to use internet sources for help if it's absolutely necessary (with proper citation); however, the solutions you turn in should be your own work and written in your own words.

Note: Please list the names of any students or faculty who you worked with on the assignment.

1. Read the article "Recent intense hurricane response to global climate change," Greg Holland and Cindy L. Bruyère, *Climate Dynamics* **42** (2014), pp. 617–627. Don't be overly concerned with all of the specifics; just try and get an overall sense of the purpose of the paper and the techniques used.

   (a) Write a paragraph or two summarizing the main findings of the research presented in the article. What questions did the authors set out to answer and how did they accomplish this?

   (b) In terms of climate modeling, what adjustments did the authors make to their models in order to address their research questions? Do you think these adjustments are fair or are they too presumptive?

   (c) Based on their modeling, the authors provide a bit of good news concerning the number of intense hurricanes expected in the future. What is this "silver lining?"

2. In class we derived the following equilibrium temperature profiles for three different Energy Balance Models. In each case we assume that the albedo function $\alpha(y) = \alpha$ is constant and that $0 < \alpha < 1$. Recall that $y = \sin\theta$ represents the "latitude" on the planet and that $0 \le y \le 1$ (assuming symmetry about the equator). Also, $s(y) = 1.241 - 0.723y^2$ is the quadratic approximation to the insolation distribution.

$$T_1(y) = \frac{1}{B}\left(Q(1-\alpha) - A\right) \qquad \text{linear OLR model} \tag{1}$$

$$T_2(y) = \frac{1}{B}\left(Qs(y)(1-\alpha) - A\right) \qquad \text{includes latitude dependence} \tag{2}$$

$$T_3(y) = \frac{Q(1-\alpha)}{B+C}\left(s(y) + \frac{C}{B}\right) - \frac{A}{B} \qquad \text{includes heat transport} \tag{3}$$

   (a) Find the exact value of y, denote it by y^*, where T_1 and T_2 intersect (see the left plot in Figure 1). This number does *not* depend on any of the parameters.

   (b) Show that the area between T_1 and T_2 to the left of y^* is equal to the area between T_1 and T_2 to the right of y^* (see the left plot in Figure 1).

   (c) Show that T_2 and T_3 also intersect at y^* (see the right plot in Figure 1).

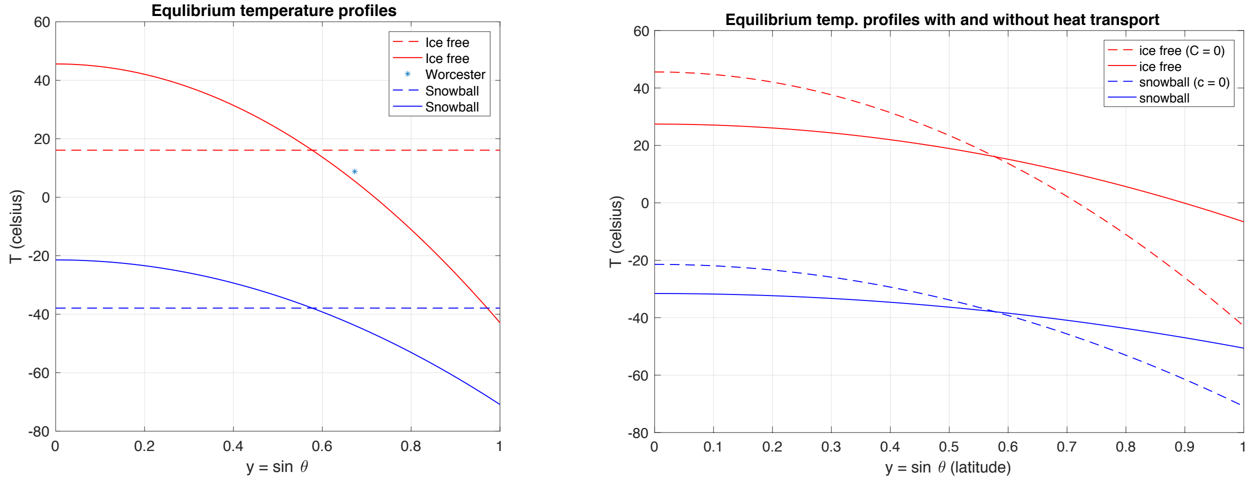

Figure 1: Equilibrium temperature profiles for different albedo values: $\alpha = 0.32$ (red); $\alpha = 0.62$ (blue). Left figure: T_1 (dashed) versus T_2 (solid). Right figure: T_2 (dashed) versus T_3 (solid).

(d) Show that the area between T_2 and T_3 to the left of y^* is equal to the area between T_2 and T_3 to the right of y^* (see the right plot in Figure 1).

(e) Recall that $\overline{T} = \int_0^1 T(y)\,dy$ gives the global mean temperature of the Earth. Show that, for a fixed choice of parameters, $\overline{T_1} = \overline{T_2} = \overline{T_3}$, that is, the average global equilibrium temperatures for each model are identical. Based on the physical differences between each model, why does this result make sense?

3. Consider the two-step albedo function with ice line η given by

$$\alpha(y;\eta) = \begin{cases} \alpha_w & \text{if } y < \eta \\ \alpha_s & \text{if } y > \eta, \end{cases}$$

where α_w represents the albedo rate for water and land, α_s is the rate for snow and ice, and $0 \le \eta \le 1$. The semi-colon is used to indicate that the albedo function depends on the value of the ice line η. Recall that $\overline{\alpha} = \int_0^1 s(y)\alpha(y)\,dy$ gives the weighted average albedo over the planet.

(a) Show that $\overline{\alpha} = \alpha_s + (\alpha_w - \alpha_s)(1.241\eta - 0.241\eta^3)$.

(b) Using the values $\alpha_w = 0.32$ and $\alpha_s = 0.62$, compute the value of $\overline{\alpha}$ (four decimal places) assuming that the ice line η is located at the latitude of:

(i) the north pole, **(ii)** Worcester, **(iii)** the Tropic of Cancer, **(iv)** the equator.