The Greenhouse Effect

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Lecture Outline

- Observations from Computer Project #1: Bifurcations and Hysteresis
- The Greenhouse Effect
- Ice Core Data and Human-Induced Climate Change
Computer Project #1: Observations

- Modeling is an iterative process. Sometimes the results are poor or counter-intuitive. Adjust your model.

- Units are important (e.g., working in kelvin or Celsius, metric system or not).

- There are typically lots of parameters ($\sigma, Q, \alpha, \epsilon$) and the outcome of a model can vary greatly even for a small change in the value of a parameter—bifurcations.

- Tuning: Sometimes we adjust the parameters to make our model agree with known data. This looks good, but can also be misleading to those evaluating the model.

**Example:** Climate Model #3 introduced $\epsilon$ to model the greenhouse effect and obtain the current average temperature of the Earth. No physics used at all: $Q(1 - \alpha) = \epsilon\sigma T^4$
Climate Model #5

\[
C \frac{dT}{dt} = E_{\text{in}} - E_{\text{out}} = (1 - \alpha(T))Q - \epsilon\sigma T^4
\]

where

\[T = \text{global average surface temperature, in K}\]

\[\alpha(T) = 0.7 - 0.4 \frac{e^{(T-265)/5}}{1 + e^{(T-265)/5}} \quad \text{(albedo)}\]

\[Q = 1/4 \text{ of the solar constant } S, 342 \text{ W/m}^2\]

\[\sigma = 5.67 \cdot 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4\)\]

\[\epsilon = \text{greenhouse effect parameter}\]
Figure: The bifurcation that arises when decreasing $\epsilon$ in Climate Model #5.
Climate Model #5: Bifurcation #1

Bifurcation value: $\epsilon_h \approx 0.488$

- For $\epsilon > \epsilon_h$, there are three equilibrium temperatures. The largest (warm, current climate) and the smallest (frigid, snowball state) are stable (sinks).

- At $\epsilon = \epsilon_h$ (a saddle-node bifurcation), the two smaller equilibria merge into one, forming a node. The larger equilibrium point has increased in value (to approximately 305 K).

- For $\epsilon < \epsilon_h$, there is only one equilibrium temperature corresponding to a very warm planet (a hothouse over 305 K $\approx 32^\circ$ C).
Figure: The bifurcation that arises when increasing $\epsilon$ in Climate Model #5.
Bifurcation value: $\epsilon_{sb} \approx 0.691$

- For $\epsilon < \epsilon_{sb}$, there are three equilibrium temperatures. The largest (warm, current climate) and the smallest (frigid, snowball state) are stable (sinks).

- At $\epsilon = \epsilon_{sb}$ (a saddle-node bifurcation), the two larger equilibria merge into one, forming a node. The smaller equilibrium point has decreased in value (to approximately 225 K).

- For $\epsilon > \epsilon_{sb}$, there is only one equilibrium temperature corresponding to a very frigid planet (Snowball Earth, less than 225 K $\approx -48^\circ$ C). Amazingly, there is evidence that Earth was in this state about 630 Mya (million years ago) and 715 Mya.

- The bifurcations at $\epsilon_{h}$ and $\epsilon_{sb}$ are examples of tipping points, where it is suddenly possible to move from one climate state to another. The bifurcation diagram demonstrates the phenomenon of hysteresis.
The Greenhouse Effect

Recall: Climate Model #2 (incorporating albedo $\alpha$ but no greenhouse effect $\epsilon = 0$) predicts the Earth’s temperature to be $254.9 \, K = -18.25^\circ C$ (snowball!) This would be the temperature of the Earth without the greenhouse effect.

The Greenhouse Effect: Greenhouse gases (carbon dioxide = CO2, water vapor = H2O, methane = CH4) are transparent to visible light, but opaque to infrared light.

The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

Who discovered the greenhouse effect? A mathematician!

A U.S. senator has called global warming the "greatest hoax ever foisted on the American people. But despite persistently strident rhetoric, skeptics are having an ever-harder time making their arguments: scientific support for warming continues to grow.

**GREENHOUSE EFFECT**

A prerequisite for life on earth, the greenhouse effect occurs when infrared radiation (heat) is retained within the atmosphere.

1. Most solar energy reaching the earth is absorbed at the surface.
2. The warmed surface emits infrared radiation.
3. Like a blanket, atmospheric greenhouse gases absorb and reradiate the heat in all directions, including back to the earth.
4. Human activity has increased the amount of greenhouse gas in the atmosphere and thus the amount of heat returned to the surface. In consequence, global temperatures have risen.

The Greenhouse Effect: Origins

- Theory well formulated by the nobel-prize winning Swedish chemist Svante Arrhenius (1859–1927).


- Made extensive calculations on the effect that increasing the amount of CO2 in the atmosphere would have on the Earth’s surface temperature—calculations that have lasted until this day. His predictions agree with those obtained from current super-computer global climate models.
Historical Overview of Climate Change Science

Chapter 1

Frequently Asked Question 1.1

What Factors Determine Earth's Climate?

The climate system is a complex, interactive system consisting of the atmosphere, land surface, snow and ice, oceans and other bodies of water, and living things. The atmospheric component of the climate system most obviously characterises climate; climate is often defined as 'average weather'. Climate is usually described in terms of the mean and variability of temperature, precipitation and wind over a period of time, ranging from months to millions of years (the classical period is 30 years). The climate system evolves in time under the influence of its own internal dynamics and due to changes in external factors that affect climate (called 'forcings'). External forcings include natural phenomena such as volcanic eruptions and solar variations, as well as human-induced changes in atmospheric composition. Solar radiation powers the climate system. There are three fundamental ways to change the radiation balance of the Earth: 1) by changing the incoming solar radiation (e.g., by changes in Earth's orbit or in the Sun itself); 2) by changing the fraction of solar radiation that is reflected (called 'albedo'; e.g., by changes in cloud cover, atmospheric particles or vegetation); and 3) by altering the longwave radiation from Earth back towards space (e.g., by changing greenhouse gas concentrations). Climate, in turn, responds directly to such changes, as well as indirectly, through a variety of feedback mechanisms.

The amount of energy reaching the top of Earth's atmosphere each second on a surface area of one square metre facing the Sun during daytime is about 1,370 Watts, and the amount of energy per square metre per second averaged over the entire planet is one-quarter of this (see Figure 1). About 30% of the sunlight that reaches the top of the atmosphere is reflected back to space. Roughly two-thirds of this reflectivity is due to clouds and small particles in the atmosphere known as 'aerosols'. Light-coloured areas of Earth's surface – mainly snow, ice and deserts – reflect the remaining one-third of the sunlight. The most dramatic change in aerosol-produced reflectivity comes when major volcanic eruptions eject material very high into the atmosphere. Rain typically reduces the reflectivity of the aerosol.

FAQ 1.1, Figure 1. Estimate of the Earth's annual and global mean energy balance. Over the long term, the amount of incoming solar radiation absorbed by the Earth and atmosphere is balanced by the Earth and atmosphere releasing the same amount of outgoing longwave radiation. About half of the incoming solar radiation is absorbed by the Earth's surface. This energy is transferred to the atmosphere by warming the air in contact with the surface (thermals), by evapotranspiration and by longwave radiation that is absorbed by clouds and greenhouse gases. The atmosphere in turn radiates longwave energy back to Earth as well as out to space. Source: Kiehl and Trenberth (1997).

Figure: Heat Balance. Recall: \[ Q = \frac{S}{4} = 342 \text{ W/m}^2. \] Source: “Historical Overview of Climate Change Science,” IPCC AR4, (2007) p. 96.
Understanding the Earth’s Past Climate

Figure: Lake Vostok, Antarctica.

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Figure: Some ice cores contain ice 800,000 years old (carbon dating). The proportions of different oxygen and hydrogen isotopes help reconstruct ancient temperatures; air trapped in bubbles can be analyzed to determine past levels of greenhouse gases.
The overall amplitude of the glacial–interglacial temperature cycles in the Vostok record. The same is true in the deep-sea record, with Vostok. We use the stacked timescale for ice on the lower axis, with indication of corresponding depths on the top axis) of: 

- CH$_4$ (ppm.v.)
- CO$_2$ (ppm.v.)
- Antarctic temp. ($T_a$)
- Mid-June insolation at 65$^\circ$N ($I_{65^\circ}$)
- Temperature above the inversion level ($T_{inv}$)

Series with respect to time (GT4), mid-June insolation at 65$^\circ$N ($I_{65^\circ}$), and temperature above the inversion level ($T_{inv}$). We calculate temperature from CO$_2$ data by a factor of $S = 0.0062$, where 

$$ T_a = \frac{S}{18} \cdot \delta^2D $$

and 

$$ T_{inv} = \frac{S}{3} \cdot \delta^2D $$

$\delta^2D$ is the globally averaged change from today's value of seawater isotopic content of snow in East Antarctica ($\delta^2D_{sw}$), and $\beta$ is the reference period for the dating method ($\beta = 1$). We use the stacked Vostok time series and insolation. Series with respect to time (GT4), CO$_2$ (ppm.v.), $\delta^18O_{atm}$ (%o), and mid-June insolation at 65$^\circ$N ($I_{65^\circ}$).

**Figure:** Time series data from ice core samples in Lake Vostok, Antarctica. a = CO$_2$, b = Antarctic temp., c = Methane. Source: “Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica,” Petit, et. al., *Nature* 399, June 3, 1999, pp. 429–436.
**Figure:** Very strong correlation between temperature and CO2 concentrations from the Vostok ice core data. Concentrations over the last 420,000 years range from 185 to 300 ppmV (parts per million by volume).
Figure: CO2 concentrations measured from the Mauna Loa Observatory in Hawaii as of January 2018. Source: NOAA Earth System Research Laboratory, Global Monitoring Division.
Cause for Concern: Human-Induced Warming

- Human activity since the Industrial Revolution (e.g., burning of fossil fuels) has led to massive increases in greenhouse gases.

- “atmospheric concentrations of carbon dioxide, methane and nitrous oxide are unprecedented in at least the last 800,000 years. Their effects, together with those of other anthropogenic drivers, have been detected throughout the climate system and are extremely likely to have been the dominant cause of the observed warming since the mid-20th century.” — IPCC Fifth Assessment Report, 2014.

- How much will the temperature rise? According to Vostok ice core data, a naive estimate is $10^\circ$C for every 100 ppmV. But how long will this take to happen?!
Summary

- Even simple low-dimensional climate models can help explain the mechanisms by which the Earth has transitioned between different states (e.g., from hothouse to snowball). **Bifurcation theory** is an important tool for the applied mathematician.

- Without greenhouse gases, the Earth would be in a frigid, uninhabitable, snowball state.

- Ice core data provides excellent information about the planet’s past climate, revealing a strong correlation between the amount of greenhouse gases in the atmosphere and the temperature.

- Over the past 420,000 years, CO2 concentrations have been in the range 185–310 ppmV. Now they are over 400 ppmV and climbing, resulting in a 1°C rise in temperature over the 20th century mean. We are responsible for this.