Complex Analysis

MATH 305, TuTh 11:00 - 12:15, Smith Labs 155, Spring 2016

Professor Gareth E. Roberts

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Office hours: Mon. 2:00 - 3:00, Tues. 10:00 - 11:00, Wed. 1:00 - 2:00, Thurs. 10:00 - 11:00 or by appointment.

Required Text: *Complex Variables and Applications*, Eighth ed., James Ward Brown and Ruel V. Churchill. (A copy of the text is on reserve in the Science Library.)

Course Prerequisite: MATH 242

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA305/homepage.html`
Homework assignments, computer projects, exam materials, handouts, useful links, and other important information will be posted at this site. Please bookmark it!

Course objectives:

- Develop an understanding of the fundamental concepts in complex analysis.
- Demonstrate the ability to perform standard computations in complex analysis.
- Become proficient at making clear and coherent mathematical arguments.
- Work, communicate, and share your knowledge with your peers.
- Have FUN learning complex analysis!

Syllabus: Complex analysis is a beautiful subject, with many surprising and profound results. This course will explore many of the concepts from one-variable calculus (limits, continuity, differentiation, integrals, power series), but applied to functions of a complex variable $z = x + iy$. Here, x and y are real numbers and i is the *imaginary* number that satisfies $i^2 = -1$. In many ways, studying the calculus of complex functions is much more natural and elegant than its real counterpart. For example, if a complex-valued function is differentiable on an open set in the complex plane, then it is actually infinitely-differentiable there. This is not the case for real-valued functions of a real variable.

While the mathematics of complex analysis is inherently appealing, there are also many important applications to real-world problems, particularly involving ordinary and partial differential equations (e.g., the Schrödinger equation). One particular application we will discuss is *Laplace's equation*, which arises in certain physical cases such as heat flow and the motion of certain kinds of waves. Complex analysis is also a useful tool in other branches of mathematics such as algebra, number theory, geometry, and dynamical systems. For example, one theorem that we will prove is *Liouville's Theorem*, which quickly leads to a straight-forward proof of the *Fundamental Theorem of Algebra*.

Our study will be both computational and analytical. There will be many exercises reminiscent of calculus (e.g., find the value of an integral or compute an infinite series expansion). There will also be rigorous proofs demonstrated in class and assigned for homework. One of the strengths of complex analysis is the manner in which many deep facts in the subject can be easily verified.

We will cover most of the material in Chapters 1 - 7 of the course text by Brown and Churchill. A rough outline of the semester is given below, with a few classes allotted for computer projects.

- Complex Numbers: complex plane, addition and multiplication, modulus, conjugate, exponential form, argument, roots (3 classes)
- Analytic Functions: mappings, limits, continuity, derivatives, Cauchy-Riemann equations, harmonic functions (6 classes)
- Exam 1
- Elementary Functions: exponential, logarithmic, branch cuts, trig functions, complex exponents (2 classes)
- Integrals: definite integrals, contour integrals, Cauchy-Goursat theorem, Cauchy integral formula, Liouville's theorem, the Fundamental Theorem of Algebra, maximum modulus principle, minimal surfaces (7 classes)
- Exam 2
- Series: convergence, Taylor and Laurent series, uniform convergence, integration and differentiation of (3 classes)
- Residues and Poles: Cauchy's residue theorem, isolated singularities (removable, poles, essential) (2 classes)
- Applications of Residues: real improper integrals, Fourier analysis (2 classes)
- Final Exam (Cumulative)

Homework: There will be regular homework assignments throughout the semester. Assignments will be posted on the course web page. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: Late homework will not be accepted.

Computer Projects: There will be a few computer projects assigned over the course of the semester involving the software package Maple. Topics to be explored include visualizing mappings of complex functions and minimal surfaces arising from soap film. Projects will be completed in groups of 2 to 3 people with one report to be turned in for the entire group.

Exams and Quizzes: There will be two midterm exams given in class on **Thursday, March 3** and on **Thursday, April 21**, as well as a comprehensive final at the end of the semester. Any conflicts with the midterm exams or final must be legitimate and brought to my attention well before the exam is scheduled. In addition, there will be 3-4 short in-class quizzes throughout the semester designed to make sure that fundamental definitions, theorems, and concepts are well understood.

If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact the director of Disability Services in Hogan 215 (x3693) to obtain documentation of your disability.

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment (or exam) and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Grade: Your course grade will be determined by the scores you receive for each of the following items:

- participation/interest 5%
- homework and computer projects 24%
- quizzes 6%
- midterm exams 40%
- final exam 25%

How to do well in this course:

- Attend class, participate, and ask questions. Be an engaged learner.
- Do your homework regularly.
- Read the text. (Yes, this is possible!)
- Work with your classmates. Organize study groups.

Never regard study as a duty, but as the enviable opportunity to learn. – Albert Einstein

$$\text{Euler's formula:} \quad e^{i\theta} = \cos\theta + i\sin\theta$$

$$\text{Cauchy integral formula:} \quad f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_0}\,dz$$

$$\text{Cauchy's residue theorem:} \quad \int_C f(z)\,dz = 2\pi i \sum_{k=1}^{n} \operatorname*{Res}_{z=z_k} f(z)$$