Computer Project #3: Energy Balance Models

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Mathematical Models
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Computer Project #3: Observations

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  Example: Climate Model #3 introduced ϵ to model the greenhouse effect and obtain the current average temperature of the Earth. No physics used at all: $Q(1 - \alpha) = \epsilon\sigma T^4$

Climate Model #5

$$C\frac{dT}{dt} = E_{\text{in}} - E_{\text{out}}$$
$$= (1 - \alpha(T))Q - \epsilon\sigma T^4$$

where

$$T = \text{global average surface temperature, in K}$$

$$\alpha(T) = 0.7 - 0.4\frac{e^{(T-265)/5}}{1 + e^{(T-265)/5}} \quad \text{(albedo)}$$

$$Q = 1/4 \text{ of the solar constant } S\text{, } 342 \text{ W/m}^2$$

$$\sigma = 5.67 \cdot 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)$$

$$\epsilon = \text{greenhouse effect parameter}$$

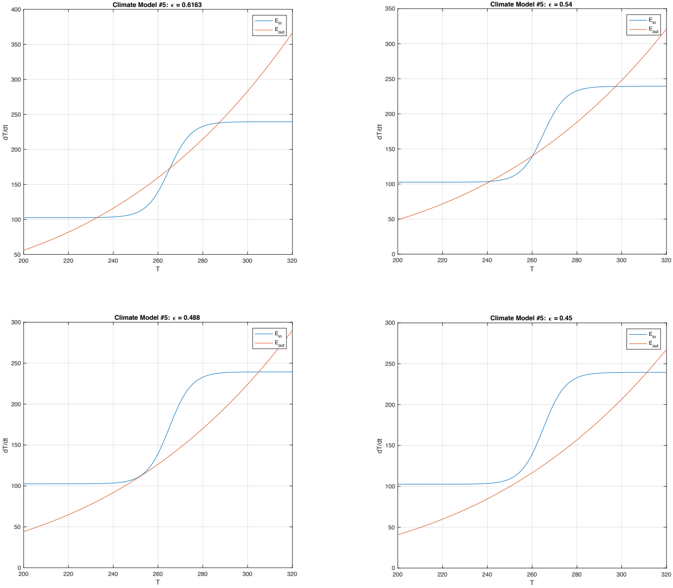

Figure: The bifurcation that arises when decreasing ϵ below $\epsilon_h \approx 0.4900676$.

Climate Model #5: Bifurcation #1

Bifurcation value: $\epsilon_h \approx 0.4900676$

- For $\epsilon > \epsilon_h$, there are three equilibrium temperatures. The largest (warm, current climate) and the smallest (frigid, snowball state) are stable (sinks).

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- At $\epsilon = \epsilon_h$ (a saddle-node bifurcation), the two smaller equilibria merge into one, forming a node. The larger equilibrium point has increased in value (to approximately 305 K).

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- For $\epsilon < \epsilon_h$, there is only one equilibrium temperature corresponding to a very warm planet (a hothouse over 305 K $\approx 32^\circ$ C). The greenhouse effect is so strong (because ϵ is small), that no ice can form on the planet.

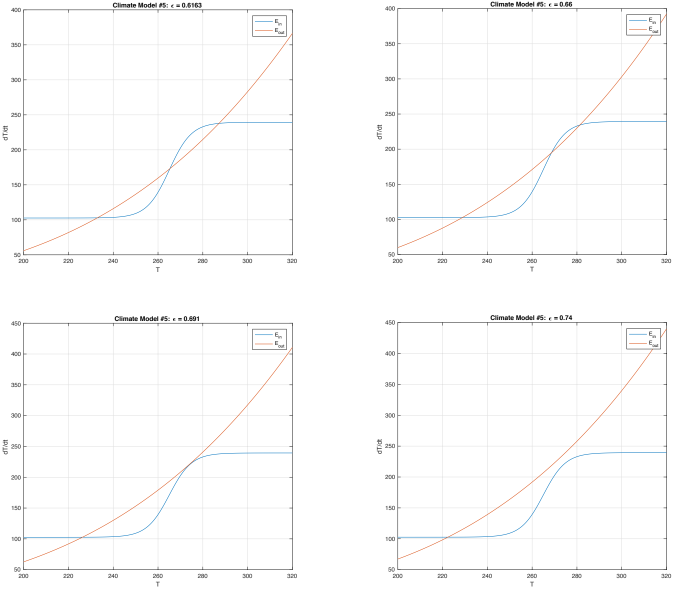

Figure: The bifurcation that arises when increasing ϵ above $\epsilon_{sb} \approx 0.6884214$.

Climate Model #5: Bifurcation #2

Bifurcation value: $\epsilon_{sb} \approx 0.6884214$

- For $\epsilon < \epsilon_{sb}$, there are three equilibrium temperatures. The largest (warm, current climate) and the smallest (frigid, snowball state) are stable (sinks).

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Bifurcation value: $\epsilon_{sb} \approx 0.6884214$

- For $\epsilon < \epsilon_{sb}$, there are three equilibrium temperatures. The largest (warm, current climate) and the smallest (frigid, snowball state) are stable (sinks).

- At $\epsilon = \epsilon_{sb}$ (a saddle-node bifurcation), the two larger equilibria merge into one, forming a node. The smaller equilibrium point has decreased in value (to approximately 225 K).

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- For $\epsilon > \epsilon_{sb}$, there is only one equilibrium temperature corresponding to a very frigid planet (Snowball Earth, less than 225 K $\approx -48^\circ$ C). Here, the greenhouse effect is so weak (because ϵ is too big), that ice envelopes the planet.

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- For $\epsilon > \epsilon_{sb}$, there is only one equilibrium temperature corresponding to a very frigid planet (Snowball Earth, less than 225 K $\approx -48^\circ$ C). Here, the greenhouse effect is so weak (because ϵ is too big), that ice envelopes the planet.

  Amazingly, there is evidence that Earth was in this state about 630 Mya (million years ago) and 715 Mya.

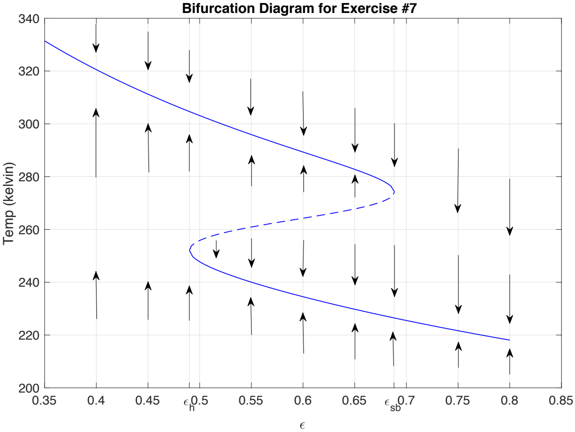

Figure: The bifurcation diagram for climate model #5 as ϵ varies. Two saddle-node bifurcations occur at $\epsilon = \epsilon_h \approx 0.49$ and $\epsilon = \epsilon_{sb} \approx 0.69$ (tipping points), demonstrating the phenomenon of hysteresis. The bifurcations suggest a mechanism for the climate to suddenly shift between vastly different steady states (e.g., from a warm, stable climate to a frigid Snowball Earth.