1. Read Chapter 3, “Modeling Change One Step at a Time,” from the course textbook *Topics in Mathematical Modeling* by K. K. Tung. What is a “chaotic” bank balance and how might it occur?

2. Consider the first-order linear difference equation
   \[ y_{k+1} = 2y_k - 1, \quad y_0 = 4. \]
   (a) Write out the first 6 terms of the sequence generated by the recursion.
   (b) Using the formula derived in class, find and simplify the solution \( y_k \). (Your answer will only depend on \( k \).) Check your formula works for the first few values of the sequence.
   (c) Suppose the initial condition is changed to \( y_0 = 1 \). Repeat parts (a) and (b). What’s different?

3. Consider the first-order linear difference equation
   \[ y_{k+1} = \frac{1}{3} y_k + \frac{2}{3}, \quad y_0 = 4. \]
   (a) Write out the first 6 terms of the sequence generated by the recursion.
   (b) Using the formula derived in class, find and simplify the solution \( y_k \). Check your formula works for the first few values of the sequence.
   (c) Compute \( \lim_{k \to \infty} y_k \).

4. Consider the general first-order linear difference equation \( y_{k+1} = ay_k + b \). Let \( c = b/(1 - a) \).
   (a) Show that if \( y_0 = c \), then the resulting sequence repeats itself forever, that is, \( y_k = c \forall k \). We call \( y_0 = c \) a **fixed point** or an **equilibrium**.
   (b) Show that if \( |a| < 1 \), then for any initial condition \( y_0 \), \( \lim_{k \to \infty} y_k = c \). In this case, \( c \) is called an **attracting fixed point** or a **sink**.

5. Working with mortgages.
   (a) Suppose you borrow $200,000 on a 15-year mortgage at an annual interest rate of 4.25% compounded monthly. What is your monthly mortgage payment? Check that your answer agrees with an online mortgage calculator.
(b) Suppose you borrow $200,000 on a 30-year mortgage at an annual interest rate of 4.75% compounded monthly. What is your monthly mortgage payment? (Typically, interest rates on a 30-year mortgage are about 0.5% higher than those for a 15-year mortgage.) Check that your answer agrees with an online mortgage calculator.

(c) Find the total amount of interest payed in parts (a) and (b) (15-year versus 30-year mortgage). How much money do you save by paying off your mortgage in 15 years?

(d) Repeat parts (a) and (b), but assume that the interest is compounded continuously.

6. Complete the following exercises from the course textbook:

  Chapter 2 (pp. 65–67): # 2, 5, 6

  Hints: For each problem, try setting up the difference equation first rather than just plugging into formulas. In some cases, you will want to measure time $t$ in months rather than years. Problem #6b is easier than it seems (it is related to #4a).