

MATH 242: Principles of Analysis

Homework Assignment #6

DUE DATE: Thurs., Oct. 29, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. In this problem you will show that the **Cantor set** is uncountable. To begin, we need to understand how to write a number in its ternary (base 3) expansion. We say that the sequence $0.a_1a_2a_3\dots$ where each $a_i \in \{0, 1, 2\}$ is a ternary expansion for the number x if

$$x = \sum_{n=1}^{\infty} \frac{a_n}{3^n}.$$

(Note that the series above converges by using the comparison test.)

For example, $x = 0$ has the ternary expansion $0.000\dots$ while $x = 1$ has the expansion $0.222\dots$ since

$$\sum_{n=1}^{\infty} \frac{2}{3^n} = \frac{2/3}{1 - 1/3} = 1$$

(geometric series with first term $2/3$ and ratio $1/3$.) The number $x = 1/3$ has two ternary expansions, either $0.1000\dots$ or $0.022222\dots$.

The key fact that you may use for this problem (without proof) is that x is in the Cantor set if and only if x has a ternary expansion containing only 0's and 2's.

- (a) Recall that in constructing the Cantor set, we removed $(1/3, 2/3)$ to create C_1 . Then, to create C_2 , we deleted $(1/9, 2/9)$ and $(7/9, 8/9)$ from C_1 . List the open intervals that are removed from C_2 to create C_3 .
 - (b) What number has a ternary expansion of $0.020202\dots$?
 - (c) Is $1/4$ in the Cantor set? Explain.
 - (d) Use the key fact above to prove that the Cantor set is uncountable.
2. Do the following exercises from the course text *Understanding Analysis* by Stephen Abbott: **3.2.1 (part (b) only), 3.2.2, 3.2.3 (parts (c), (d) and (e)), 3.2.12, 3.3.2, 3.3.3, 3.3.5, 3.3.7, 3.4.7 (part (a) only)**

Note: For problem #3.3.5, decide which set is compact using the Heine-Borel Theorem (ignore the directions in the text.)