MATH 241-02 Multivariable Calculus

SOLUTIONS to Sample Final Exam Questions

1. **(a)** **(ii)**
   (b) **(i)**
   (c) **(iii)**
   (d) **(ii)**
   (e) **(iii)**
2. **(a)** $\overrightarrow{AB} = <1, -2, 2>$, $\overrightarrow{AC} = <2, 0, 2>$, $\overrightarrow{BC} = <1, 2, 0>$
   (b) B and C
   (c) $45°$ or $\pi/4$
   (d) $x = 5 - 4t, y = 2t, z = 1 + 4t$
3. $\mathbf{T}(1) = <\frac{1}{3}, \frac{2}{3}, \frac{2}{3}>, \mathbf{N}(1) = <-\frac{2}{3\sqrt{5}}, -\frac{4}{3\sqrt{5}}, \frac{5}{3\sqrt{5}}>$
4. **(a)** $<\cos t, \sin t, t>$, 3 loops
   (b) speed is $\sqrt{2}$, time to snitch is 6π
   (c) 6π meters
   (d) $9\pi/\sqrt{2}$, Gryffindor wins (of course!)
5. **(a)** Domain is $\mathbb{R}^2$, Range is $3 \leq z \leq 5$
   (b) $(0, 0)$ is a saddle point.
   (c) Level curves are hyperbolas. The x- and y-axes are level curves.
6. **(a)** $(0, 0), (1, 1), (-1, -1)$.
   (b) $(0, 0)$ is a saddle point, $(1, 1)$ is a local maximum, $(-1, -1)$ is a local maximum.
   (c) No absolute min, but absolute max is 0.
7. 64 (use Lagrange multipliers).
8. **(a)** 0
   (b) $\frac{16\sqrt{2}\,\pi}{3}\left(2 - \sqrt{3}\right)$
9. **(a)** Show that $Q_x - P_y = 0$.
   (b) $f(x, y) = e^{xy} + \sin(x - y) + 3y$
   (c) $1 - 3\pi/2$
10. 0

11. **(a)** $0 \le r < \infty$, $0 \le \theta \le \pi/2$. The value of the integral is $\pi/4$.

(b)

$$\begin{aligned}
\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy &= \int_0^\infty \int_0^\infty e^{-x^2} \cdot e^{-y^2}\,dy\,dx \\
&= \int_0^\infty e^{-x^2}\,dx \cdot \int_0^\infty e^{-y^2}\,dy \\
&= \left(\int_0^\infty e^{-x^2}\,dx\right)^2 .
\end{aligned}$$

(c) $\left(\int_0^\infty e^{-x^2}\,dx\right)^2 = \pi/4 \quad \Longrightarrow \quad \int_0^\infty e^{-x^2}\,dx = \sqrt{\pi}/2.$