MATH 136-03 Calculus 2, Spring 2019

Sample Final Exam Questions

Below are some **sample** final exam questions from past exams. **Note:** Collectively, these are not intended to represent an actual exam nor do they completely cover all the material that could be asked on the exam.

1. Define $A(x) = \int_0^x f(t) dt$ for $0 \le x \le 5$, where the graph of f(t) is given below.



- (a) Find A(0), A(3) and A(5).
- (b) Find A'(1) if it exists. If it does not exist, explain why.
- (c) Find A''(1) if it exists. If it does not exist, explain why.
- (d) Find the intervals on which A(x) is increasing and decreasing.
- (e) Find the intervals on which A(x) is concave up and concave down.
- (f) Sketch a graph of A(x) over the interval $0 \le x \le 5$.
- 2. Evaluate the following integrals:

(a)
$$\int \sqrt{2x+1} + \sin(3x) dx$$

(b)
$$\int \sec^2(2\theta) e^{\tan(2\theta)} d\theta$$

(c)
$$\int t^6 \ln t dt$$

(d)
$$\int \frac{z^2}{\sqrt{1-z^2}} dz$$

(e)
$$\int \frac{6}{x(x-1)(x+1)} dx$$

- 3. Approximate the value of the integral $\int_{1}^{3} \cos(x^2) dx$ using the given rule:
 - (a) Midpoint Rule M_4
 - (b) Trapezoid Rule T_4
 - (c) Simpson's Rule S_8

4. Let R be the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = x^2$.

- (a) Sketch the region R and find its area.
- (b) Find the volume of the solid of revolution obtained by rotating R about the x-axis.
- (c) Find the volume of the solid of revolution obtained by rotating R about the line x = -2.
- 5. Sequences and Series:
 - (a) Find a formula for the general term a_n (start with n = 1) for the sequence

$$-1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \frac{1}{32}, -+\dots$$

- (b) Does the sequence given by $a_n = \ln\left(\frac{1+en^3}{4+n^3}\right)$ converge or diverge? If it converges, find the limit.
- (c) Find the sum of the geometric series: $18 6 + 2 2/3 + 2/9 + \cdots$
- (d) Find the interval of convergence of the infinite series $\sum_{n=1}^{\infty} \frac{x^n}{n^2 4^n}$.
- 6. Determine whether the given infinite series converges or diverges using any of the tests from class or the text. You must provide a valid reason to receive full credit.

(a)
$$\sum_{n=2}^{\infty} \frac{n^3}{n^4 - 1}$$

(b) $\sum_{n=1}^{\infty} \frac{\cos n}{n^3}$
(c) $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$
(d) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$
(e) $\sum_{n=1}^{\infty} ne^{-2n}$

7. Find the solution to the given initial-value problems:

(a)
$$\frac{dy}{dt} = \frac{y}{1+t^2}, \quad y(0) = 3.$$

(b)
$$\frac{dy}{dx} = -2x^2e^y$$
, $y(3) = -\ln 9$.

8. Consider the initial-value problem

$$\frac{dy}{dt} = 2(t+1)y^2, \quad y(0) = 1/2.$$

- (a) Using Euler's method with a step-size of $\Delta t = 0.2$, approximate the value of the solution when t = 1. In other words, approximate y(1) where y(t) is the solution to the given ODE.
- (b) Solve the ODE with the given initial condition.
- (c) Using your answer to part (b), calculate y(1). What is the error in your approximation? (Any ideas to why it is so far off?)
- 9. Suppose that Auntie Pat is cooking her Thanksgiving turkey (tofurkey for you vegetarians) for friends and family. The guests are planning to arrive at 5:00 pm. She preheats the oven to 400°F. Suppose the initial temperature of the turkey is 50°F. She places the turkey in the oven at 10:00 am. By noon the turkey has cooked to a temperature of 80°F. Using Newton's law of cooling (or warming), at what time (to the nearest minute) will the temperature of the turkey be 150°F (medium rare and ready to serve)? Assume that the oven has a constant temperature of 400°F throughout the cooking. Does she make it in time for the guests or will she be serving hors d'ouvres for a while?
- 10. Some conceptual questions:

(a) If
$$F(x) = \int_{x^2}^{5} \cos(\sqrt{t} + \pi) dt$$
, find $F'(\pi)$.

- (b) Derive the formula for the volume of a sphere of radius r by rotating the top half of the circle $x^2 + y^2 = r^2$ about the x-axis.
- (c) Find the average value of the function $g(x) = x \sin x$ over the interval $0 \le x \le \pi$.
- (d) Suppose that p(x) is a piecewise function defined as follows:

$$p(x) = \begin{cases} 0 & \text{if } x < 0 \text{ or } x > 2\\ Cx^2(2-x) & \text{if } 0 \le x \le 2. \end{cases}$$

Find the value of C which makes p a probability density function.

(e) TRUE or FALSE: If true, provide a brief explanation. If false, give a counterexample to the statement.

If $\lim_{n \to \infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

- (f) Find the sum of the infinite telescoping series:
 - $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \cdots$