MATH 134 Calculus 2 with FUNdamentals

Applications to Economics: Consumer and Producer Surplus

SOLUTIONS

Exercise 1: Suppose that the demand function for producing a can of tennis balls is $p(x) = 20 - 0.05x$ and the supply function is $s(x) = 2 + 0.0002x^2$.

(a) Find the equilibrium price $\overline{p}$ and quantity $\overline{x}$ by solving $p(x) = s(x)$.

Answer: First solve the equation $p(x) = s(x)$ to find $\overline{x}$. We have

$$\begin{aligned} p(x) = s(x) \implies \quad 20 - 0.05x &= 2 + 0.0002x^2 \\ \implies \quad 0.0002x^2 + 0.05x - 18 &= 0 \\ \implies \quad x^2 + 250x - 90{,}000 &= 0 \\ \implies \quad (x - 200)(x + 450) &= 0, \end{aligned}$$

which implies $\overline{x} = 200$, since it must be a positive value. Then, we compute $p(200) = s(200) = 10$ so that the equilibrium price is $\overline{p} = \$10$.

(b) Find the value of the consumer surplus and producer surplus at the equilibrium price.

Answer: Using the integral formula for consumer surplus, we find that

$$\text{CS} = \int_0^{200} (20 - 0.05x) - 10\, dx = \int_0^{200} 10 - 0.05x\, dx = 10x - 0.025x^2\Big|_0^{200} = \$1{,}000\,.$$

Using the integral formula for producer surplus, we find that

$$\text{PS} = \int_0^{200} 10 - (2 + 0.0002x^2)\, dx = \int_0^{200} 8 - 0.0002x^2\, dx = 8x - \frac{0.0002}{3}x^3\Big|_0^{200} = \$1{,}066.67\,.$$

(c) Suppose that the price is set to \$1 **greater** than the equilibrium price. Find the new value of the consumer and producer surplus and check that the total surplus (CS+PS) is less than the value obtained in part **(b)**. Use the same $\overline{x}$ in each integral (i.e., solve $p(x) = \overline{p} + 1$ first to obtain $\overline{x}$ and then use this as a limit of integration in both integrals).

Answer: If we set the new price to be \$11, then the new quantity demanded at that price is found by solving $p(x) = 11$. This yields $20 - 0.05x = 11$ or $0.05x = 9$. Thus $x = 180$ is our new value for $\overline{x}$. The consumer surplus at the new price is given by

$$\text{CS} = \int_0^{180} (20 - 0.05x) - 11\, dx = \int_0^{180} 9 - 0.05x\, dx = 9x - 0.025x^2\Big|_0^{180} = \$810\,.$$

Notice that this value is less than the consumer surplus at the equilibrium price because the higher price leads to a lower consumer benefit.

The producer surplus at the new price (using the same value for $\overline{x}$) is given by

$$\text{PS} = \int_0^{180} 11 - (2 + 0.0002x^2)\, dx = \int_0^{180} 9 - 0.0002x^2\, dx = 9x - \frac{0.0002}{3}x^3\Big|_0^{180} = \$1{,}231.20\,.$$

Notice that this value is larger than the producer surplus at the equilibrium price because the higher price leads to greater profits for the producers.

However, the new total surplus is \$810 + \$1,231.20 = \$2,041.20, which is less than \$2,066.67, the value of the total surplus at the equilibrium price. This agrees with the economic theory that the maximum total surplus occurs at the intersection of the supply and demand curves. The loss in surplus to the economy (in this case \$25.47) is called the **deadweight loss**.

Exercise 2: Repeat the instructions from Exercise 1 using $p(x) = \dfrac{15}{2x+1}$ and $s(x) = 4x + 3$. In this case, assume that x is measured in thousands of cans and the price p (and s) is measured in dollars per can.

Answer: **(a)** First solve the equation $p(x) = s(x)$ to find $\overline{x}$. We have

$$\begin{aligned} p(x) = s(x) \quad &\Longrightarrow \quad \frac{15}{2x+1} \;=\; 4x+3 \\ &\Longrightarrow \quad 15 \;=\; (2x+1)(4x+3) \\ &\Longrightarrow \quad 8x^2 + 10x - 12 \;=\; 0 \\ &\Longrightarrow \quad 4x^2 + 5x - 6 \;=\; 0 \\ &\Longrightarrow \quad (x+2)(4x-3) \;=\; 0, \end{aligned}$$

which implies $\overline{x} = 3/4$, since it must be a positive value. Then, we compute $p(3/4) = s(3/4) = 6$ so that the equilibrium price is $\overline{p} = \$6$.

(b) Using the integral formula for consumer surplus, we find that

$$\text{CS} \;=\; \int_0^{3/4} \frac{15}{2x+1} - 6\,dx \;=\; \frac{15}{2}\ln|2x+1| - 6x\Big|_0^{3/4} \;=\; \frac{15}{2}\ln(5/2) - \frac{9}{2} - 0 \;=\; 2.372.$$

The first term in the integral is computed using the u-sub $u = 2x + 1$, $du = 2\,dx$. Since x is measured in thousands of cans and p is in dollars per can, we multiply this value by 1,000 to obtain the total value of the consumer surplus, \$2,372.

Using the integral formula for producer surplus, we find that

$$\text{PS} \;=\; \int_0^{3/4} 6 - (4x+3)\,dx \;=\; \int_0^{3/4} 3 - 4x\,dx \;=\; 3x - 2x^2\Big|_0^{3/4} \;=\; \frac{9}{4} - \frac{9}{8} \;=\; \frac{9}{8} \;=\; 1.125.$$

As with the consumer surplus, we multiply this value by 1,000 to obtain the total value of the producer surplus, \$1,125.

(c) If we set the new price to be \$7, then the new quantity demanded at that price is found by solving $p(x) = 7$. This yields $\frac{15}{2x+1} = 7$ or $15 = 14x + 7$. Thus $x = 4/7$ is our new value for $\overline{x}$. The consumer surplus at the new price is given by

$$\text{CS} \;=\; \int_0^{4/7} \frac{15}{2x+1} - 7\,dx \;=\; \frac{15}{2}\ln|2x+1| - 7x\Big|_0^{4/7} \;=\; \frac{15}{2}\ln(15/7) - 4 \;=\; 1.716.$$

Since x is measured in thousands of cans and p is in dollars per can, we multiply this value by 1,000 to obtain the total value of the consumer surplus, \$1,716. Notice that this value is less than the consumer surplus at the equilibrium price because the higher price leads to a lower consumer benefit.

The producer surplus at the new price (using the same value for $\overline{x}$) is given by

$$\text{PS} \;=\; \int_0^{4/7} 7-(4x+3)\;dx \;=\; \int_0^{4/7} 4-4x\;dx \;=\; 4x-2x^2\Big|_0^{4/7} \;=\; \frac{16}{7}-\frac{32}{49} \;=\; \frac{80}{49} \;=\; 1.633.$$

As with the consumer surplus, we multiply this value by 1,000 to obtain the total value of the producer surplus, \$1,633. Notice that this value is larger than the producer surplus at the equilibrium price because the higher price leads to greater profits for the producers.

However, the new total surplus is \$1,716 + \$1,633 = \$3,349, which is less than \$3,497, the value of the total surplus at the equilibrium price. This agrees with the economic theory that the maximum total surplus occurs at the intersection of the supply and demand curves.