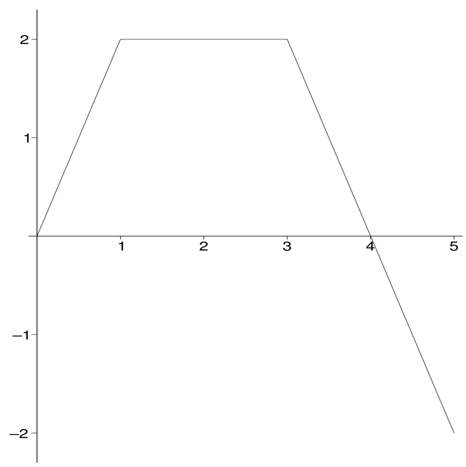
# MATH 134-02 Sample Final Exam Questions

**SPRING 2014** **Prof. Gareth Roberts**

Below are some **sample** final exam questions. **Note:** Collectively, these are not intended to represent an actual exam nor do they completely cover all the material that could be asked on the exam.

1. Define $F(x) = \int_0^x f(t)\,dt$ for $0 \le x \le 5$, where the graph of $f(t)$ is given below.




   **(a)** Find $F(0)$, $F(3)$ and $F(5)$.

   **(b)** Find $F'(1)$ if it exists. If it does not exist, explain why.

   **(c)** Find $F''(1)$ if it exists. If it does not exist, explain why.

   **(d)** Find the intervals on which F(x) is increasing and decreasing.

   **(e)** Find the intervals on which F(x) is concave up and concave down.

   **(f)** Sketch a graph of $F(x)$ over the interval $0 \le x \le 5$.

2. Evaluate the following integrals.

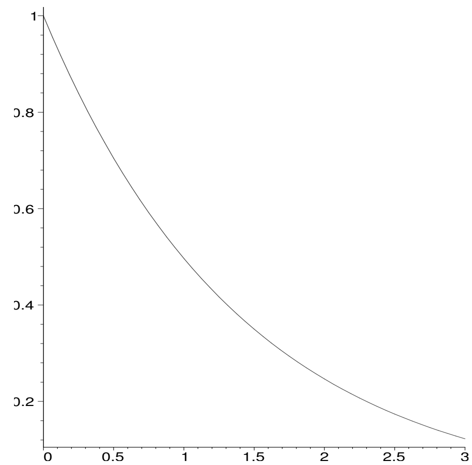
   **(a)** $\int \sqrt{2x+1} + \sin(3x)\,dx$

   **(b)** $\int \sec^2(2\theta)\,e^{\tan(2\theta)}\,d\theta$

   **(c)** $\int x^6 \ln x\,dx$

   **(d)** $\int \frac{\sqrt{4y^2-13}}{y^2}\,dy$ Use the formula $\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln|u + \sqrt{u^2-a^2}\,| + c.$

   **(e)** $\int \frac{6}{x(x-1)(x+1)}\,dx$




3. Consider the function $g(x)$ whose graph is shown above:

   The following numbers are the left, right, trapezoid and midpoint approximations to $\int_0^3 g(x)\,dx$, each with $n = 50$ subdivisions.

   (I) 1.227491645 (II) 1.253541536 (III) 1.253817953 (IV) 1.280144260

   **(a)** Match each value with one of the four approximations, explaining your choices.

   **(b)** Use Simpson's rule to approximate the value of the integral.

4. Let $R$ be the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = x^2$.

   **(a)** Sketch the region $R$ and find its area.

   **(b)** Find the volume of the solid of revolution obtained by rotating $R$ about the $x$-axis.

   **(c)** Find the volume of the solid of revolution obtained by rotating $R$ about the $y$-axis.

5. The demand function for a given product is $p = 66 + \frac{384}{x+2}$ and the supply curve is $s = 4x + 58$.

   **(a)** Find the equilibrium price $\bar{p}$ and the equilibrium quantity $\bar{x}$.

   **(b)** Find the consumer surplus and the producer surplus.

6. Find the solution to the given initial-value problems:

   **(a)** $\dfrac{dy}{dt} = \dfrac{y}{1+t^2}, \quad y(0) = 3.$

   **(b)** $\dfrac{dy}{dx} = y^{2014} - 1, \quad y(0) = 1.$

7. Consider the initial-value problem

$$\frac{dy}{dx} = 2(x+1)y^2, \quad y(0) = 1/2.$$

**(a)** Using Euler's method with a step-size of $\Delta x = 0.2$, approximate the value of the solution when $x = 1$. In other words, approximate $y(1)$ where $y(x)$ is the solution to the given ODE.

**(b)** Solve the ODE with the given initial condition.

**(c)** Using your answer to part **(b)**, calculate $y(1)$. What is the error in your approximation? (Any ideas to why it is so far off?)

8. Suppose that Auntie Pat is cooking her Thanksgiving turkey (tofurkey for you vegetarians) for friends and family. The guests are planning to arrive at 5:00 pm. She preheats the oven to 400°F. Suppose the initial temperature of the turkey is 50°F. She places the turkey in the oven at 10:00 am. By noon the turkey has cooked to a temperature of 80°F. Using Newton's law of cooling (or warming), at what time (to the nearest minute) will the temperature of the turkey be 150°F (medium rare and ready to serve)? Assume that the oven has a constant temperature of 400°F throughout the cooking. Does she make it in time for the guests or will she be serving hors d'ouvres for a while?

9. Sequences and Series:

**(a)** Find a formula for the general term $a_n$ for the sequence

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, + - \dots$$

**(b)** Does the sequence given by $a_n = \frac{1-n^3}{4+3n^3}$ converge or diverge? If it converges, find the limit.

**(c)** Find the sum of the given geometric series: $18 - 6 + 2 - 2/3 + 2/9 - + \cdots$

**(d)** Does the given infinite series converge or diverge? Identify which test you are using and explain how you arrived at your answer.

(i) $\displaystyle\sum_{n=2}^{\infty} \frac{3n^4}{2n^5+1}$ (ii) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{\pi^n}$ (iii) $\displaystyle\sum_{n=1}^{\infty} ne^{-2n}$

10. Some conceptual questions:

**(a)** Derive the formula for the volume of a sphere of radius $r$ by rotating the top half of the circle $x^2 + y^2 = r^2$ about the $x$-axis.

**(b)** Find the exact length of the curve parametrized by the equations $x = 5 - 3t^2, y = 1 + 2t^3$ from $t = 0$ to $t = 2\sqrt{2}$.

**(c)** Find the average value of the function $x \sin x$ over the interval $0 \leq x \leq \pi$.

**(d)** Suppose that $f(x)$ is a piecewise function defined as follows:

$$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ \dfrac{k}{x^3} & \text{if } x \geq 1. \end{cases}$$

Find the value of $k$ which makes $f$ a probability density function (pdf).