MATH 133 Calculus 1 with FUNdamentals

Practice Exam 3

1. Calculate the derivative of each function. **Simplify** your answer as best as possible.

   (a) $f(x) = \dfrac{3}{x^3} + 3^x + e^3$

   (b) $g(x) = (\tan x + x^3)^6$

   (c) $F(t) = t^2 e^{\sqrt{4t+1}}$

   (d) $h(x) = \tan^{-1}(\sin(5x))$

   (e) $G(\theta) = \ln(\, \cos(3\theta)\,)$

2. For the equation below, use implicit differentiation to calculate dy/dx.

$$xy^2 + e^{y^3} \;=\; \tan^{-1} x - \cos y$$

3. Find the absolute maximum and absolute minimum of the function

$$g(\theta) \;=\; \theta - 2\sin\theta$$

   over the interval $[0, 2\pi]$. Give the maximum and minimum function values (exact numbers as well as decimals rounded to three places), and the θ-values (in radians) where they occur.

4. Let $f(x) = x^5 - 15x^3$. Find and classify all of the critical points (local max, local min, or neither). Find the inflection points. Use the first and second derivatives to sketch a graph of the function.

5. Wire of length 12 m is divided into two pieces and each piece is bent into a square. How should the wire be divided in order to minimize the sum of the areas of the squares? Check that your answer is really a minimum.

   Hint: Let x and y represent the side lengths of each square, respectively.

6. **Calculus Potpourri:**

   (a) The total dollar cost of producing x high-definition television sets is given by the function

$$C(x) = 300 - 100x - 0.2x^2 + 0.002x^3.$$

   Find the marginal cost function and use it to *estimate* the cost of producing the 251st television set.

   (b) Suppose that $G(x) = f(x^2)$ and that $f'(9) = 1/2$. Find $G'(3)$.

   (c) Use L'Hôpital's Rule to compute $\displaystyle\lim_{x\to 0} \frac{\cos(3x) - 1}{5x^2}$.

   (d) Find the vertical and horizontal asymptotes (if they exist) of the function $g(x) = \dfrac{4x^2 - 1}{x^2 - 9}$.