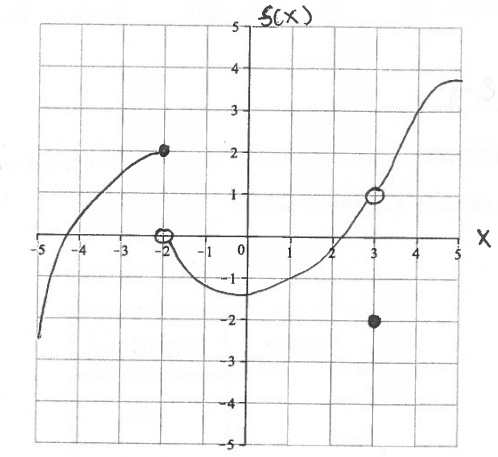
MATH 133 Calculus 1 with FUNdamentals

Practice Exam 2

1. The graph of $f(x)$ is shown below. Use it to answer the following questions.



   (a) State the precise mathematical definition for a function $f(x)$ to be **continuous** at the point $x = a$.

   (b) Using your definition from part **(a)**, give a mathematical explanation as to why f is not continuous at $x = -2$.

   (c) Is f left-continuous, right-continuous, or neither at $x = -2$? Explain.

   (d) Evaluate each of the following:

   (i) $\lim_{x \to 3} f(x)$ **(ii)** $f(3)$

   (e) How should $f(3)$ be **redefined** to remove the discontinuity at $x = 3$?

2. Evaluate each of the following limits, if they exist. Note that ∞ or $-\infty$ are acceptable answers. You must show work (e.g., algebra) to receive full credit.

   (a) $\lim_{x \to \pi} \sqrt{2\cos x + 5}$

   (b) $\lim_{x \to 4} \dfrac{2x^2 - 7x - 4}{x^2 - 16}$

   (c) $\lim_{\theta \to 0} \dfrac{\cos(3\theta) \cdot \sin\theta}{\theta}$

   (d) $\lim_{x \to 2} \left(\dfrac{1}{2(x-2)} - \dfrac{2}{x^2 - 4} \right)$

   (e) $\lim_{x \to \infty} \tan^{-1} \left(\dfrac{1 + x^4}{3x^3 - 4x^2 + 5} \right)$

3. Use the Intermediate Value Theorem to prove that the equation $e^x = \cos(2x) + 1$ has a solution in the interval $[0, \pi/4]$.

4. **(a)** State the two limit definitions for the derivative of a function $f(x)$ at the point $x = a$.

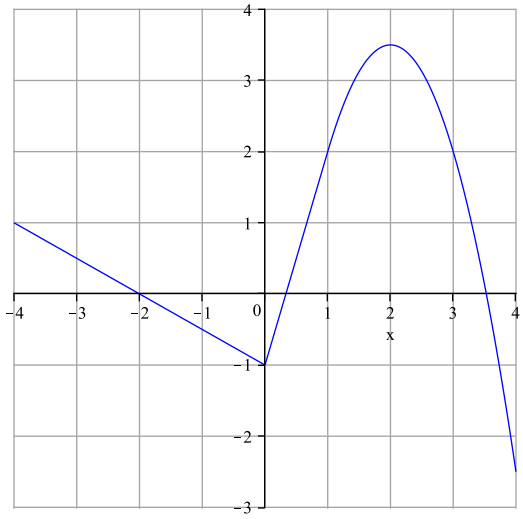
$$f'(a) =$$

or

$$f'(a) =$$

(b) Using one of your limit definitions from part **(a)**, find $f'(2)$ if $f(x) = \sqrt{5x-1}$.

5. The graph of $f(x)$ is shown below. Determine (estimate if necessary) the following values of the derivative:



(a) $f'(-2)$

(b) $f'(0)$

(c) $f'(1/2)$

(d) $f'(2)$

(e) $f'(3)$

(f) Sketch a graph of $f'(x)$.

6. **Calculus Potpourri:** You must show your work to receive any partial credit.

(a) If $F(x) = x^2 \cos x$, find and simplify $F''(x)$.

(b) If $6x - 8 \leq g(x) \leq 3x^2 - 5$ for all x, find $\lim_{x \to 1} g(x)$.

(c) Find any horizontal asymptotes for the function $y(t) = \dfrac{8t - 5t^3 + 9t^7}{\pi + 4t^5 - 3t^7}$.

(d) Find and simplify $F'(x)$ if $F(x) = 5e^x - \dfrac{3}{x^5} + \pi^3$.

(e) Find the equation of the tangent line (in slope-intercept form) to $f(x) = 6\sqrt{x} - \frac{1}{4}x^2$ at the point $a = 4$.

(f) Sketch a graph of a function $g(x)$ that satisfies $g'(x) > 0, g''(x) < 0$ and $\lim_{x \to \infty} g(x) = 4$.