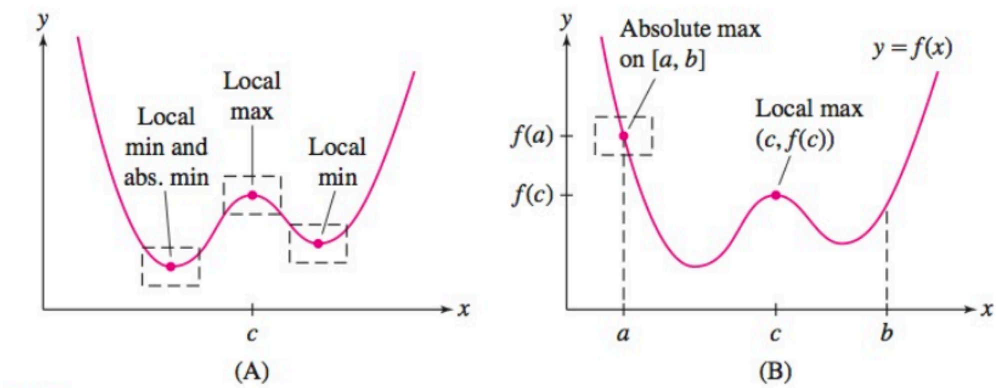
MATH 133 Calculus 1 with FUNdamentals

Section 4.2: Extreme Values

This section is concerned with finding the maximum or minimum values of a continuous function over a closed interval. One common application of calculus is to **optimize** a given function. For example, an analyst working for a company wants to find the optimal price for maximizing profit, or an engineer seeks to minimize the cost of the material for making a solar battery. These are problems requiring techniques for finding the maximum or minimum of a function.

Absolute Versus Local Extrema:

The **absolute maximum** of a function is the largest value of the function over its *entire* domain. A **local maximum** is the largest value of a function over a small neighborhood, but not necessarily over the whole domain. For example, you might me the fastest person in the class (local max), but not the fastest at the College (absolute max). A similar definition applies to the **absolute minimum** or **local minimum** of a function, except now we seek the *smallest* values. The figure below demonstrates the differences between local and absolute max's and min's.



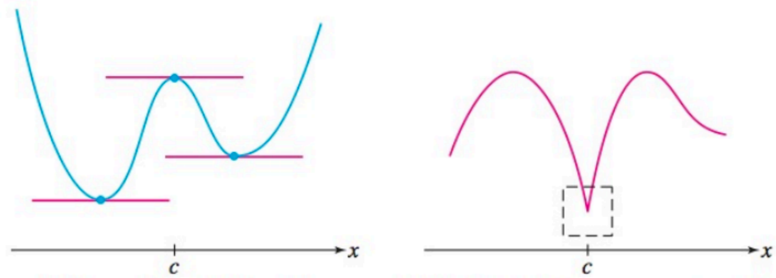
Exercise 1: Using a graph, state the absolute max and min of each function.

(a) $f(x) = x^2 + 2$ **(b)** $g(x) = 4\sin x$ **(c)** $h(x) = x^3$

Note that in some cases the absolute max or min is not attained (∞ or $-\infty$); however, if we restrict the domain of a continuous function to a closed interval $[a, b]$, then the absolute max or min always occurs somewhere on the interval, possibly at an endpoint (see figure (B) above). This fact is called the Extreme Value Theorem.

Extreme Value Theorem (EVT): If f is a continuous function on the interval $[a, b]$, then f attains an absolute maximum and an absolute minimum for some x-values in $[a, b]$.

Our goal is to find the absolute max and min of a continuous function over a closed interval. You will notice from the figure below, that local max's and min's occur at places where the derivative is zero or possibly at a location where the derivative does not exist. These key points are called **critical points**.



(A) Tangent line is horizontal at the local extrema.

(B) This local minimum occurs at a point where the function is not differentiable.

> **Definition:** $x = c$ is called a **critical point** of $f(x)$ if either
>
> **(a)** $f'(c) = 0$, or
>
> **(b)** $f'(c)$ does not exist.

Finding the absolute max and min of a $f(x)$ over $[a, b]$: Here are the steps:

1. Find all critical points c by solving the equation $f'(x) = 0$. Look out for places where f' does not exist.

2. Find the value of f at the endpoints, that is, compute $f(a)$ and $f(b)$.

3. Compare the values of $f(c)$ for all critical points with $f(a)$ and $f(b)$, and pick the largest (absolute max) and smallest (absolute min).

Exercises: Find the absolute max and min of the function over the given interval. You should do these problems on a separate piece of paper.

2. $f(x) = x^3 - 9x^2 - 48x + 52$ on the interval $[-5, 14]$.

3. $g(x) = \dfrac{x}{x^2 + 4}$ over the interval $[0, 3]$.

4. $h(x) = \sin x + \cos^2 x$ over the interval $[0, 2\pi]$.