MATH 133 Calculus 1 with FUNdamentals

Section 4.6: Graph Sketching and Asymptotes

Key Idea: The point of this section is to combine all of the information obtained from the first and second derivatives (intervals where the function is increasing, decreasing, concave up or down, critical points, and inflection points) and use it to draw a graph of the function. Horizontal and vertical asymptotes (if they exist) are also important to locate when sketching a graph.

Exercise 1: Consider the function $f(x) = x^6 - 6x^4$. Find and classify all critical points. Find any inflection points. Use the first and second derivatives to sketch the graph of f.

Functions with Asymptotes:

Recall that $f(x)$ has a **vertical asymptote** at $x = b$ if either $\lim_{x \to b^+} f(x) = \pm\infty$ or $\lim_{x \to b^-} f(x) = \pm\infty$. Typically, vertical asymptotes are found by finding where the **denominator** of a rational function equals 0. A function $f(x)$ has a **horizontal asymptote** at $y = k$ if $\lim_{x \to \infty} f(x) = k$ or $\lim_{x \to -\infty} f(x) = k$. In the first case, the asymptote occurs to the far right of the graph, while in the second case, the asymptote appears to the far left.

Exercise 2: Identify the vertical and horizontal asymptotes of the function $F(x) = \dfrac{3x^2 + 5x - 7}{4x^2 - 100}$.

Exercise 3: Identify the vertical and horizontal asymptotes of $g(x) = \dfrac{5x-3}{2x+1}$ and then use the first and second derivatives of g to sketch its graph.

Exercise 4: Carefully find and simplify the first and second derivatives of $f(x) = xe^{-x}$. Use this information to find the critical and inflection points of f. Sketch the graph of f. Are there any asymptotes? **Hint:** Use L'Hôpital's Rule.