MATH 133 Calculus 1 with FUNdamentals

Section 3.4: Rates of Change

Below we consider some applications of the derivative from the fields of economics and climate science. The key idea to remember is that dy/dx (the derivative of y with respect to x) measures the instantaneous **rate of change** of y with respect to x. So, for example, if $T(t)$ measures the temperature T (in degrees Celsius) of an object as a function of time t (in seconds), then dT/dt gives the rate of change of the temperature of the object. The units of dT/dt are $°C$/sec. If $dT/dt > 0$, then the object is warming; if $dT/dt < 0$, then the object is cooling.

Physics: If $s(t)$ is the position of a moving object (or particle on a line) as a function of time t, then $s'(t) = ds/dt = v(t)$ is the instantaneous **velocity** and $s''(t) = v'(t) = a(t)$ is the **acceleration**. The speed of the object is defined to be $|s'(t)| = |v(t)|$, which is always positive.

Economics: Let $C(x)$ represent the cost of producing a quantity x of some item. For example, $C(25) = \$3{,}000$ means it costs \$3,000 to produce 25 of the particular item. The derivative $C'(x)$ is called the **marginal cost**. It tells us approximately how much it costs to produce the next item, the $(x+1)$st item. Similarly, if $P(x)$ is the profit made form selling x items, then $P'(x)$ is called the **marginal profit**, and if $R(x)$ is the revenue made form selling x items, then $R'(x)$ is called the **marginal revenue**. The **average cost** of producing x items is given by $\frac{C(x)}{x}$.

Exercise 1: Suppose $C(x) = 8000 - 10x + x^2 + 0.01x^3$ represents the cost of producing x computers.

a) What is the cost of producing 10 computers?

b) Find the marginal cost function.

c) Find $C'(10)$ and explain its meaning. What are the units of $C'(10)$?

d) Find the actual cost of producing the 11th computer. Compare your answer with $C'(10)$.

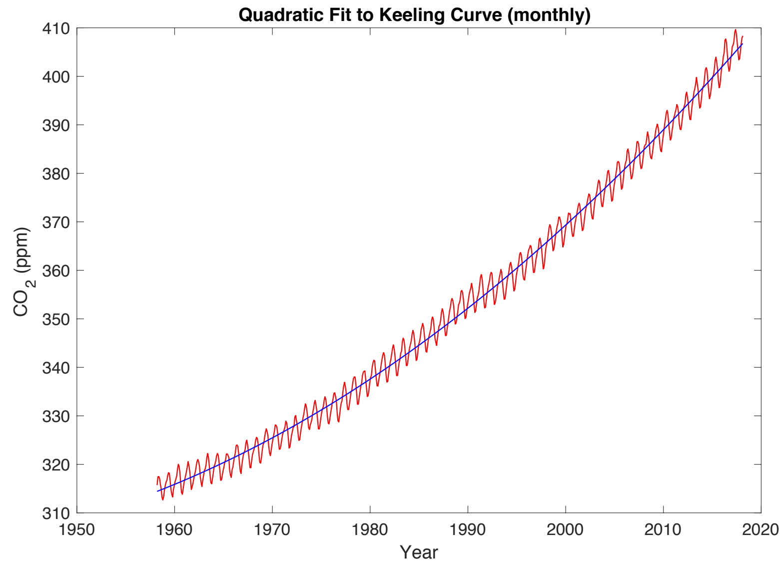

Figure 1: The Keeling curve (shown in red) is the amount of carbon dioxide (measured in parts per million) present at the Mauna Loa Observatory as a function of time (in years). The blue curve is a quadratic approximation to the Keeling curve.

Climate Science: Figure 1 shows a plot of the famous *Keeling curve*, one of the most important data sets in climate science. Measurements of carbon dioxide (CO_2) at the Mauna Loa Observatory at the Mauna Loa volcano in Hawaii were begun by Charles Keeling in March of 1958 and continue to this day under the guidance of his son Ralph Keeling. Air samples have been taken hourly, every day, using the same measuring technique for over 60 years.

To put these numbers in perspective, the highest previous concentration of CO_2 in the last 800,000 years was 300 ppm. The last time the planet had CO_2 amounts this high was over 3 million years ago, when temperatures were roughly 5°F warmer and sea level was 65 feet higher!

Exercise 2: The function $M(t) = 0.01258t^2 - 48.465t + 46{,}995$ is an approximation to the Keeling curve, where t is measured in years.

a) Compute $M'(t)$ and $M''(t)$.

b) Find $M(2020)$ and $M'(2020)$. Explain the meaning of these two numbers.

c) Explain the significance of the sign of $M''(t)$. How would you explain it to a layperson (e.g., your congressional representative)?

d) Estimate the amount of CO_2 at Mauna Loa in 2050 and 2100.

e) Notice that the Keeling curve (in red) oscillates as it rises. Why do you think this occurs? **Hint:** What is the period of these oscillations?