MATH 110: Mathematics and Music
Homework Assignment #6
DUE DATE: Wed., March 28, start of class.

You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the assignment.

Limerick - kciremiL

First, let me explain that I'm cursed;
I'm a poet whose time gets reversed.
Reversed gets time
Whose poet a I'm;
Cursed I'm that explain me let, first.

1. Listen to the CD *Math and Music 3: Temperament and Musical Group Theory,* distributed in class on March 23. Liner notes for the CD are available from the course webpage. You may be tested on some of this music so be sure to read the liner notes and listen carefully. One approach to effectively absorbing the music is to make brief notes about each piece, listing significant details as you listen.

2. Read Chapter 6 of the course text, *The geometry of music* by Wilfrid Hodges. Some of this material was covered in class at the start of the semester.

3. On page 99, Hodges gives five different classes of symmetries, *p1, ph, pv, p2, phv.* For each of the letters **B**, **Λ**, **L**, **T** and **Θ**, give the corresponding symmetry class.

4. Give two examples of music using symmetry *p2.* What type of symmetry does Mozart use in his *Clarinet quintet, K381*?

5. In the key of B♭, write out the first eight measures of *Mary Had a Little Lamb* after applying an inversion about the note B♭, staying in the same key. Do this in the treble clef with the correct key signature. (You do not need to include or invert the text.) See HW#2, question 12 for the original music.

6. In the key of B♭, write out the first eight measures of *Mary Had a Little Lamb* after applying a retrograde-inversion about the note B♭, staying in the same key. Do this in the treble clef with the correct key signature.

7. Play the original and each of the two transformed melodies on the piano. How do the new melodies sound?

8. **Closure:**

   a. Which of the following sets are closed under addition? Explain.
   (i) The integers **(ii)** The rationals **(iii)** The irrationals

   b. Which of the following sets are closed under multiplication? Explain.
   (i) The integers **(ii)** The rationals **(iii)** The irrationals

9. Finish completing the group multiplication table (started in class) for the symmetries of the square. This set is called the **Dihedral group of degree 4**, denoted D_4.

10. Notice that the completed multiplication table shows that the symmetries of the square are closed under composition. In other words, applying two symmetry operations to the square successively yields one of the eight symmetries. Associativity follows from the definition of composition of functions and the identity element is contained in the set of symmetries. List the inverse of each of the eight elements of D_4 and conclude that D_4 is a group of order 8.

11. What do you notice about any row or any column in the multiplication table?

12. Is the group D_4 commutative? In other words, is it true that $a * b = b * a$ for **every** element a, b in the group D_4?

13. Using your table, in general, what is the composition of two rotations? of two reflections? of a rotation and a reflection?