College of the Holy Cross, Spring Semester, 2021
Math 241 (Professor Hwang)
Worksheet 2, Due March 31 at 5 PM

Work in pairs or groups of three; turn in only one write-up per group.

Exercise 1. Consider the function

$$\begin{aligned} f(x,y) &= x^3 - 2x^2 + x - xy^2 \\ &= x(x+y-1)(x-y-1) \end{aligned}$$

Carefully sketch the zero level set of f. Find and plot the critical points of f, then use the Hessian test to classify each as a local maximum/minimum or saddle point. Finally, sketch several level curves qualitatively.

Exercise 2. Use Lagrange multipliers to find the extreme values of f when restricted to the unit circle $g(x,y) = x^2 + y^2 = 1$. Watch for (and handle) division by zero. There are six points of interest; identify them in your sketch.

Exercise 3. Use 1. and 2. to find the extreme values of f on the closed unit disk $x^2+y^2 \leq 1$.