Work in pairs or groups of three; turn in only one write-up per group. Calculate the indicated derivatives and verify the chain rule.

**Exercise 1.** $\mathbf{R} \xrightarrow{g} \mathbf{R}^2 \xrightarrow{f} \mathbf{R}^2$: $g(t) = \begin{bmatrix} t \\ t \end{bmatrix}$, $f(r, \theta) = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$.

(a) $(f \circ g)(t) =$

(b) $D(f \circ g)(t) =$

(c) $Df(r, \theta) =$

(d) $Df\big(g(t)\big) =$

(e) $Dg(t) =$

(f) $Df\big(g(t)\big)Dg(t) =$

**Exercise 2.** $\mathbf{R}^2 \xrightarrow{g} \mathbf{R}^2 \xrightarrow{f} \mathbf{R}$: $g(u, v) = \begin{bmatrix} u^2 - v^2 \\ 2uv \end{bmatrix}$, $f(x, y) = x + y^2$.

(a) $(f \circ g)(u, v) =$

(b) $D(f \circ g)(u, v) =$

(c) $Df(x, y) =$

(d) $Df\big(g(u, v)\big) =$

(e) $Dg(u, v) =$

(f) $Df\big(g(u, v)\big)Dg(u, v) =$

**Exercise 3.** $\mathbf{R}^2 \xrightarrow{g} \mathbf{R}^2 \xrightarrow{f} \mathbf{R}^2$: $g(r, \theta) = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$ $f(x, y) = \begin{bmatrix} x^2 - y^2 \\ 2xy \end{bmatrix}$.

(a) $(f \circ g)(r, \theta) =$

(b) $D(f \circ g)(r, \theta) =$

(c) $Df(x, y) =$

(d) $Df\big(g(r, \theta)\big) =$

(e) $Dg(r, \theta) =$

(f) $Df\big(g(r, \theta)\big)Dg(r, \theta) =$