My Mathematical Thesis

A. Student

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Abstract

An abstract is a paragraph or few that gives the reader an overview of the document. Abstracts are commonly found on research articles, but not necessarily on theses.

1 Introduction

This document illustrates some capabilities of the typesetting system LaTeX. To a surprisingly large extent, you only need to type the words; LaTeX handles formatting. LaTeX *commands* are text strings that follow a backslash, such as `\LaTeX` (for the distinctively-typeset logo) or `\emph{}` (which emphasizes the text between the braces). Certain characters are special; to get #, $, %, or & in text, precede them with a backslash.[1]

1.1 Further Subdivision

You will probably begin writing your thesis with an outline. When you type your thesis in LaTeX, you use sectioning commands to reflect the logical structure of your document.

A new paragraph is begun by placing a blank line in the input file. Multiple spaces in a LaTeX file are not significant. LaTeX recognizes when a space signifies the end of a sentence, and adjusts the appearance accordingly. To fine-tune the spacing and appearance of your document, you should first type the entire file, then tweak if you must. While you may be accustomed to word processors, whose primary emphasis is *appearance*, you should try to think in terms of organization and good writing when using LaTeX.

As a beginner, you should avoid worrying about appearance; type design and page composition are sophisticated enterprises requiring years of study. Instead, concentrate on the quality and logical structure of your document:

[1] In LaTeX, the dollar sign toggles math mode (Section 2.1), while the percent sign is a comment character. The pound sign and ampersand have more advanced meanings, related to macro parameters and alignment, respectively.

good writing, overall organization, efficient and flexible use of LaTeX macros, neatly formatted input. Concepts and notation that appear frequently in your document require carefully designed macros. Your thesis advisor is a good person to consult about this issue.

Unnumbered Matter

Subsection 1.1 was started with a `\subsection` command. An unnumbered subsection is created by appending an asterisk, as in `\subsection*`. The argument of a (sub)section command is the title of that portion of text, such as "Further Subdivision".

2 Mathematics

Most of a typical document is entered in *paragraph mode*. In the source file, the input is plain text, written in an ordinary text editor (not a word processor). Mathematics is subject to different rules of font choice, spacing, and line breaking than ordinary text, and is placed into special environments.

2.1 Nuts and Bolts

Short snippets of mathematics, such as $y = |f(x)|$ or $\langle \mathbf{e}_i, \mathbf{e}_j \rangle = \delta_{ij}$, are placed in a paragraph by enclosing them with dollar signs. More complicated expressions and equations, such as fractions, sums, and integrals, are often better set in a displayed equation:

$$\int_\Sigma K\,dA = 2\pi\chi_\Sigma, \qquad K\,dA = -\sqrt{-1}\left(\frac{\partial\bar{\partial}g}{g} - \frac{\partial g}{g} \wedge \frac{\bar{\partial}g}{g}\right).$$

Mathematical typesetting is an art, see [2], [3], and below. Basic points include proper use of fonts, attention to spacing, and proper line breaking. For basic documents, the LaTeX defaults work well, but as your skill progresses you should pay attention to the fine points.

Greek letters are obtained by typing their name as a command in math mode: `$\alpha$` gives α. Mathematical symbols are generated similarly in math mode. If you need a non-standard symbol, please consider carefully whether it would be better to use an English phrase instead of a symbol. For example, it is strongly preferable to write out "for every" and "there exists" instead of relying on $\forall$ and $\exists$. There are over 2000 symbols available, see [4].

Multi-line displayed equations are best handled with the AMS's enhancements, see [1]:

$$x(t) = c_1\cos(\sqrt{5}t) + c_2\sin(\sqrt{5}t) \tag{2.1}$$

$$x'(t) = \sqrt{5}\big(-c_1\sin(\sqrt{5}t) + c_2\cos(\sqrt{5}t)\big) \tag{2.2}$$

Some alignment capabilities are shown for illustration; don't fine-tune your typesetting until your thesis nears completion.

Environments for definitions, theorems, etc., are defined in this file. Section 5 illustrates their use.

3 Typographical Pro Tips

A typeset page is composed of abstract entities known as *boxes* (typographical elements) and *glue* (stretchable white space). A box may itself contain sub-boxes and glue. At the lowest level are boxes containing one symbol. Knuth's explanations, [2], are definitive and reader-friendly. A few of the most basic points are included here.

3.1 Spacing

In ordinary text, *inter-word space* occurs between adjacent words and *end-of-sentence space* occurs after the period at the end of a sentence. The size of a particular inter-word or end-of-sentence space depends slightly on how much text must fit into one line, but end-of-sentence space is about twice as large as inter-word space, and the two look awkward if used in each others stead.

LaTeX places end-of-sentence space after a non-capital letter followed by a period followed by white space. "After an abbreviation," Prof. Calculus explained, "you must tell LaTeX explicitly to use inter-word space by putting a backslash-space immediately after the period."

"Thus, `Prof. Calculus` (see above) should be typed `Prof.\ Calculus`," Prof. Calculus continued.

The converse situation—a sentence ending with a capital letter (and a period)—is rare, but can occur even if you do not work at NASA. By default, LaTeX puts inter-word space after a capital letter followed by a period. To ensure end-of-sentence space, put `\@` before the period, as in `NASA\@.` One's first impulse, to put two (or more) spaces in the input file, unfortunately has no effect whatsoever.

3.2 Line Breaks

Sometimes you want to prevent LaTeX from breaking the line at a particular point. For example, this happens before the number in Section_3, page_42, Theorem_3.1.416, or between A._Person's initial and last name, I._M. Typeset's initials, or between a noun and its mathematical symbol: the function_f, evaluated at_x. (In each case, the line break subtly interrupts the reader's concentration.) A "tie", the tilde character, gives a non-breaking space: See `page~42`. Do not leave actual space before or after a tie.

3.3 Hyphens and Dashes

Word-break hyphens needed to justify the right margin are added automatically by LaTeX. For ordinary hyphens, use a single hyphen character `-` with no surrounding space.

Two consecutive hyphen characters (with no surrounding space) give an *en dash*, punctuation suitable for numerical ranges, see pp. 42–43.

Three consecutive hyphen characters (with no surrounding space) give an *em dash*, punctuation suitable—you see?—for small asides and the like.

Four or more consecutive hyphen characters give combinations of dashes and hyphens, and should never be used.

3.4 Mathematics

In mathematics, most spaces in the input are ignored. One em of space (roughly the width of a lower-case m) may be obtained with `\quad`; two ems may be obtained with `\qquad`. Both commands work in paragraph mode as well as math.

Though mathematical symbols are italicized, math mode is $different$ from text italics, and should not be used as a substitute.

Mathematics sometimes calls for fussy spacing: Compare $sinx$ and $\sin x$, or ydx and $y\,dx$. The first issue is avoided by using only commands for named functions: `\sin`, `\cos`, `\ln`, and so forth. These commands not only set the name upright, but they automatically add appropriate spacing between the function name and the following variable.

For the second spacing issue, sometimes you have to tweak manually. The command `\,` (backslash-comma) gives a thin space; thus, `$y\, dx$`. (The space character is not needed by LaTeX, but makes the input easier for a human to read.)

3.5 Semantic Coding

In LaTeX, you can define your own commands, or *macros*. These should be fairly easy to type, and used to separate the meaning of your code from the way it will be typeset.

Notation for the real numbers provides a classic example. Before LaTeX, journals used boldface capitals, $\mathbf{R}$, $\mathbf{C}$, $\mathbf{Z}$, etc. After LaTeX, "blackboard bold" fonts were created: $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Z}$. Which should you use?

Under no circumstances should you type `\mathbf{R}` or `\mathbb{R}` each time you refer to the real numbers in a document. Instead, use indirection and abstraction to your benefit. In your preamble, define macros such as:

```
\newcommand{\Number}[1]{\mathbf{#1}}
\newcommand{\R}{\Number{R}}
\newcommand{\C}{\Number{C}} % etc.
```

In the document body, use only `\R` to denote the real numbers, `\C` for the complex numbers, etc.

Why does this matter? First, the shorter commands save considerable typing. Second, and more importantly, the two-stage coding separates the *meaning* or *semantics* (the symbol for numbers) from the typographical *appearance* or *presentation* (the font choice). If, after typing your thesis, your advisor insists you use ordinary boldface, you can convert your sources by making one trivial change (the definition of the `\Number` macro) in the preamble.

4 Further Information

LaTeX easily handles most typesetting needs of scholarly papers, including tables, figures, footnotes, cross-referencing, running headers, bibliographies, tables of contents, and indices. Some of these features are built into the standard document classes, others are provided by packages, which are explicitly imported in the preamble. Output can be converted to a number of printable and electronically publishable formats, such as Postscript, HTML, or hyperlinked PDF using widely available tools.

Once you have an idea of the LaTeX basics, you'll learn fastest by typing a document of your own, see Section 1.1. Links to further information may be found on the Holy Cross Math/CS Club's web page, under Mathematical Typesetting:

`http://math.holycross.edu/~ahwang/math_club/software.html`

If you're using OS X or GNU/Linux, you can use `ePiX` to put line diagrams into your document. Please see

`http://math.holycross.edu/~ahwang/current/ePiX.html`

for details.

5 A Short Document

Let $f : \mathbf{R} \to \mathbf{R}$ be a function.

Definition 5.1. We say f is *differentiable* at x_0 if

$$f'(x_0) := \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \tag{5.1}$$

exists. If f is differentiable at x_0 for every $x_0 \in \mathbf{R}$, we say f is *differentiable*.

Theorem 5.2. *Let f be differentiable at x_0. Then f is continuous at x_0.*

Proof. We must show that $f(x_0 + h) \to f(x_0)$ as $h \to 0$. However, by Equation (5.1) and properties of limits, we have

$$\lim_{h \to 0} f(x_0 + h) - f(x_0) = \lim_{h \to 0} h \cdot \frac{f(x_0 + h) - f(x_0)}{h} = 0 \cdot f'(x_0) = 0.$$

This establishes the theorem. □

Remark 5.3. A function may be differentiable at x_0, yet be discontinuous at x for all $x \neq x_0$.

References

[1] *User's Guide for the* `amsmath` *Package*, Version 2.0, published by the American Mathematical Society. (See the Math/CS Club page, or web search for `amsldoc.pdf`)

[2] D. E. Knuth, *The TEX Book*, Addison-Wesley, 1986.

[3] F. Mittelbach, S. Goossens, et al., *The LATEX Companion*, Second Edition, Addison-Wesley, 2004.

[4] Scott Pakin, *The Comprehensive LATEX Symbol List*. (Web search for the title.)