Unfortunately, there were several things wrong with problem 32 in Chapter 6 of the text (on Problem Set 5 for the course), and also with the first corrected version (!) (There must be something about this problem!) Here is a corrected version of the problem, incorporating some hints that should help you get started toward a solution.

A) This is different from the original “corrected version”(!) Show that if $B_m$ is the board with $m$ unshaded squares in locations 

$$(1,1), (1,2), (2,2), (2,3), \ldots,$$

in a diagonal “staircase pattern” containing exactly $m$ squares, then the rook polynomial of $B_m$ is given by:

$$R(B_m,t) = 1 + \binom{m}{1} t + \binom{m-1}{2} t^2 + \binom{m-2}{3} t^3 + \cdots + \binom{m-k+1}{k} t^k + \cdots$$

(Note: $B_{2n-1}$ is the “upper part” of the shaded squares in the board in the book problem there are $m = n + n - 1 = 2n - 1$ X’s. The cases $B_m$ with $m = 2n - 1$ odd fit on $n \times n$ boards; the cases with $m = 2n$ even don’t quite fit on square boards if the top of the staircase occurs in row 1, column 1.)

B) Use part A and our theorems on rook polynomials to show that the rook polynomial of the complement $\overline{B}$ of the $n \times n$ board $B$ given in the book problem is

$$R(\overline{B},t) = \sum_{k=0}^{n} \frac{2n}{2n-k} \binom{2n-k}{k} t^k$$

C) Use part B to find the number of ways of placing $n$ non-attacking rooks on the unshaded squares in the book’s $n \times n$ board $B$.

Comment: This problem is a version of the “hostess problem”: In how many ways can $n$ married couples be seated at a round table so that no husband sits next to his own wife. Do you see the connection? (Think of a round table, but ignore the circular symmetry!)