1. Suppose \( f = \sum_{n=1}^{\infty} f_n \), where \( f_n \) is a sequence of nonnegative measurable functions. Show that
\[
\int f = \sum_{n=1}^{\infty} \int f_n.
\]

2. Let \( f_n \) be a sequence of \( L^1 \) functions such that \( \sum_{n=1}^{\infty} \| f_n \|_{L^1} < \infty \). Show that the series \( \sum_{n=1}^{\infty} f_n \) converges (in \( L^1 \) norm) to some \( f \in L^1 \).

   Hint: Look at the proof of the Weierstrass \( M \)-Test.

3. Let \( a_n \) be an enumeration of the rational numbers in \([0, 1]\). Consider the function
\[
f(x) = \sum_{n=1}^{\infty} \frac{1}{2^n(x-a_n)^{2/3}},
\]
for \( x \in [0, 1] \).

   (a) Show that \( f \) is unbounded on every subinterval of \([0, 1]\).

   (b) Show that \( f \) is discontinuous at every point in \([0, 1]\).

   (c) Show that \( f \in L^1([0, 1]) \). Hint: Use the result of Problem 2.

4. Evaluate the expression
\[
\lim_{n \to \infty} \int_0^{\infty} \frac{n \sin(x/n)}{x(1+x^2)} \, dx
\]
as follows.

   (a) Compute
\[
f(x) = \lim_{n \to \infty} \frac{n \sin(x/n)}{x(1+x^2)}
\]
for \( x > 0 \).

   (b) Use the Dominated Convergence Theorem to prove that the expression above must equal \( \int_0^{\infty} f \).

   (c) Evaluate \( \int_0^{\infty} f \).