Math 132: Calculus for the Physical & Life Sciences 2  
Spring 2005  
Maple Lab 2: Differential Equations  
Professor Levandosky

Instructions: First, find a partner, log on to a computer and start up Maple. Be sure to actually log in (do not select workstation only), so you can save your work. Each group will turn in a single lab report with answers to the Lab Questions 2-4 below. Lab reports are due Friday, May 6.

Purpose: In this lab, you will look at the differential equation

\[
\frac{dy}{dt} = 6y(1 - y/60) - h,
\]

which models logistic population growth with harvesting. The constant \( h \) is the harvesting rate.

Lab Questions.

1. (a) Plot the slope field for \( h = 50 \). To do this, type

```
with(DEtools):
DEplot(diff(y(t),t)=6*y(t)*(1-y(t)/60)-50,[y(t)],t=0..2,y=0..100);
```

(b) In class we saw that for \( h = 50 \) there are two equilibria, \( y = 10 \) and \( y = 50 \). Plot these solutions by typing (all on one line)

```
DEplot(diff(y(t),t)=6*y(t)*(1-y(t)/60)-50,[y(t)],t=0..2,y=0..100,
[[y(0)=10],[y(0)=50]],linecolor=black);
```

(c) By modifying the line above, include the graphs of the solutions with \( y(0) = 5 \), \( y(0) = 15 \) and \( y(0) = 80 \). From these graphs it should be clear that \( y = 50 \) is a stable equilibrium and \( y = 10 \) is unstable.

2. Now consider the case \( h = 80 \).

   (a) Plot the slope field, together with several solutions.

   (b) There should be two equilibria. What are they? Are they stable or unstable?

   (c) Which initial populations survive in the long run?

3. Now consider the case \( h = 100 \).

   (a) Plot the slope field, together with several solutions. You should notice that all solutions eventually go to zero.

   (b) Now suppose that harvesting at this rate is scheduled to continue only until time \( t = 1 \). By trial and error, determine which initial populations will survive until time \( t = 1 \).

4. When \( h = 80 \) there are two equilibria and when \( h = 100 \) there are none. Somewhere between 80 and 100 there is a critical harvesting rate \( h_c \), below which there are two equilibria, and above which there are none. Find \( h_c \). Hint: Use the quadratic formula to find the roots of \( 6y(1 - y/60) - h = 0 \). When are there two roots, and when are there none?