General Information

The exam will be held THURSDAY, MARCH 31, from 6:00pm to 7:30pm in O’Neil 112. It will cover material from Sections 7.5, 7.6, 7.7, 8.1, 8.2 and 8.3 in the book. However, the material in these sections requires a thorough understanding of all of the techniques of integration covered previously. A copy of the table of integrals from the book will be provided.

Topics

• Numerical Integration. Know how to calculate LEFT, RIGHT, MID, TRAP and SIMP. Know which give underestimates or overestimates in terms of the slope and/or concavity of the graph.

• Improper Integrals. Be able to recognize when an integral is improper (when one of the limits of integration is infinite or when the function becomes infinite on the interval of integration). Know how to express improper integrals as limits, and how to evaluate these limits to determine whether a given integral converges or diverges.

• Riemann Sums and Integrals. Know how to set up Riemann sums which lead to integrals representing area, volume, arclength, etc. This includes, for example, the area between two curves.

• Cavalieri’s Principle. Know how to express the volume of a region as an integral of its cross-sectional areas.

• Arclength. Know the formulas for the arclength of a graph or parametric curve.

• Densities and Center of Mass. Know how to find the total amount of a quantity using its density function. Know the formulas for the center of mass of an object.

Practice Problems

Try working through several of the Chapter Review problems from Chapters 7 and 8. Here are a few additional problems.

1. (a) The function \( y = f(x) \) has graph shown below.

   \[ \int_0^1 f(x) \, dx \]

   The approximations \( \text{LEFT}(n) \), \( \text{RIGHT}(n) \), \( \text{MID}(n) \), and \( \text{TRAP}(n) \) to \( \int_0^1 f(x) \, dx \) are 1.23, 1.34, 1.36, 1.45 (in some order). Which method gave which value? Explain how you know which is which.
(b) Compute \( \text{LEFT}(2), \text{RIGHT}(2), \text{MID}(2), \text{TRAP}(2), \) and \( \text{SIMP}(2) \) approximations for \( \int_1^2 \frac{1}{x} \, dx \).

(c) Compute the exact value of the integral in (b), and the errors for the Trapezoidal and Simpson’s Rule approximations.

2. Determine whether each integral converges or diverges. If it converges, find its value.
   
   (a) \( \int_1^\infty \frac{dx}{x^2 + 4} \).
   
   (b) \( \int_0^5 \frac{du}{u^2 - 16} \).

3. Let \( R \) be the region bounded by \( y = 1 + x^2, x = 0, x = 1, \) and the \( x \)-axis.
   
   (a) Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.
   
   (b) Find the volume of the solid obtained by rotating \( R \) about the line \( y = -3 \).

4. Let \( R \) be the region bounded by \( y = xe^{-x}, y = 0, x = 0 \) and \( x = 4 \).
   
   (a) Sketch the region.
   
   (b) A solid has the region \( R \) as base and cross-sections perpendicular to the \( x \)-axis that are squares (extending the full width of the base). Find the volume.
   
   (c) Set up, but do not evaluate the integral to find the volume of the solid obtained if \( R \) is rotated about the \( x \)-axis.
   
   (d) Set up, but do not evaluate the integral to find the volume of the solid obtained if \( R \) is rotated about the line \( y = -2 \).

5. A wire in the shape of the graph \( y = x^2, x \in [-1, 2] \) has density \( \delta(x) = 4 - x \) at the point \( (x, x^2) \).
   
   (a) What is the total length of the wire? (Arc length!)
   
   (b) Set up a Riemann sum approximating the total mass of the wire, and explain how you got it.
   
   (c) What definite integral computes the total mass?
   
   (d) Evaluate your integral.

6. A thin metal plate has the shape of the region in the plane bounded by \( y = \sin(x), y = 0, x = \pi/4, \) and \( x = \pi/2 \). The density of the material of the plate at all points \( x \) units from the left-hand edge is \( \delta(x) = x \) grams per unit area.
   
   (a) Compute the total mass of the plate.
   
   (b) Set up (but do not evaluate) the integral to compute the \( x \)-coordinate of the center of mass.
   
   (c) From physical intuition, should the \( y \)-coordinate of the center of mass of this plate be \( \bar{y} = 1/2 \)? Why or why not?