Math 132: Worksheet on Geometric Series

1. A tortoise challenges Achilles to a race. Achilles can run 10 times faster than the tortoise, so the tortoise is given a head start, say 100 meters. In the time it takes Achilles to reach the tortoise’s starting position, the tortoise will have gone another 10 meters. By the time Achilles travels this extra 10 meters, the tortoise will have travelled an additional 1 meter. By the time Achilles travels this additional meters, the tortoise will have travelled an additional 10 centimeters, and so on. Therefore Achilles can never catch the tortoise! Use geometric series to resolve the paradox.

2. Steve and Julie are eating a stir-fry meal. They each take portions from the pan.
   
   (a) First suppose Steve takes half of the food, and Julie takes half of what remains. For seconds, Steve takes half of what is left, and then Julie takes half of what remains, and so on. What proportion of the entire meal do Steve and Julie’s portions approach?
   
   (b) Now suppose Steve begins by taking a proportion \( x \) \((0 < x < 1)\) of the food, and Julie takes half of what remains. Then Steve again takes proportion \( x \) of what remains, and Julie then takes half of what remains, and so on. What should the proportion \( x \) be so that Steve and Julie eat the same amount?

3. The Koch snowflake is obtained by taking the limit of the sequence of figures below:

   ![Koch Snowflake](image)

   (a) Find the area of the Koch snowflake. (Call the area of the triangle in the first figure \( A \).)
   
   (b) Find the perimeter of the Koch snowflake. (Call the length of the lint segments in the first figure \( L \).)

4. Suppose you take out a 30 year mortgage of $100,000.00 with an annual interest rate of 6% compounded monthly. You make monthly payments of some amount \( A \). In order for the entire mortgage to be paid off in exactly 30 years, what should \( A \) be?