Introduction.

The purpose of this lab is to use Maple to further your understanding of parametric curves.

Instructions.

Find a partner and log onto a machine. Be sure to log onto the network so you can save the worksheet to your P: drive. Answer each of the lab questions below in the Maple worksheet. To enter text into a Maple worksheet, place the cursor where you want to enter the text, click on Insert from the top menu bar, and then click on Text. When you are finished, be sure to log off the machine.

Lab Reports.

Each group will hand in a single report. The lab report will consist of a printout of your Maple worksheet, which should including all your answers to the lab questions, together with the corresponding graphs. Before you print, please reduce the size of each graph to save on paper. Lab reports are due Wednesday, November 3.

Maple Commands. The only Maple command you will need for this lab is the plot command. To plot, for example, the parametric curve

\[ x = \cos(t), y = \sin(t), \frac{\pi}{2} \leq t \leq \frac{3\pi}{2} \]

type

\[
\text{plot([cos(t),sin(t),t=Pi/2..3*Pi/2],scaling=constrained);}
\]

You should see the left half of the unit circle.

Lab Questions.

1. (a) Plot the curve \( x = 2 + 3 \cos(t), y = 1 + 3 \sin(t) \) for \( 0 \leq t \leq 2\pi \). You should see a circle. What is its center? What is its radius?

(b) How would you parametrize a circle of radius \( r \) centered at \((x_0, y_0)\)?

(c) Use your answer to part (b) to plot the circle of radius 2 centered at \((-1,1)\).

2. Plot the line segment from \((3,2)\) to \((-1,1)\).

3. Plot the curve \( x = t - \sin t, y = 1 - \cos t \) for \( 0 \leq t \leq 6\pi \). Find the values of \( t \) at which the velocity is zero. Where on the curve are these points located?

This curve is called a cycloid, and it represents the path traced out by a point on the rim of a rolling wheel.
4. \( x(t) = 2t/(t^2 + 1) \), \( y(t) = (t^2 - 1)/(t^2 + 1) \) for \(-100 \leq t \leq 100\). (You may find it useful to include the option `numpoints=100`.) Identify this curve visually, and use algebra to show that your guess is correct.

5. Try graphing \( x(t) = e^{kt} \cos t \), \( y(t) = e^{kt} \sin t \) over the interval \(-4 \leq t \leq 4\) for a few values of \( k \) (both positive and negative). How do the curves with \( k > 0 \) differ from those with \( k < 0 \)? What happens as \( k \to 0 \)?

Curves in this family are known as logarithmic spirals, and have the property that they meet every radial line through the origin at a constant angle. This property explains their appearance in biological growth. They are also used in the design of spring loaded camming devices used in rock climbing.

6. (a) Plot \( x = \cos(2t) \cos(t) \), \( y = \cos(2t) \sin(t) \) for \( 0 \leq t \leq 2\pi \). Now plot \( x = \cos(4t) \cos(t) \), \( y = \cos(4t) \sin(t) \) for \( 0 \leq t \leq 2\pi \). Without plotting, how many “petals” would you guess the curve \( x = \cos(5t) \cos(t) \), \( y = \cos(5t) \sin(t) \) has?

(b) Now plot \( x = \cos(5t) \cos(t) \), \( y = \cos(5t) \sin(t) \) for \( 0 \leq t \leq 2\pi \). Was your guess correct? Plot the same curve for \( 0 \leq t \leq \pi \). Try to explain what is going on.

These curves are often called rosettes, for the obvious reason that they look like stylized flowers.

7. Consider a circular ring of inner radius \( R \), inside of which is rolled a wheel of radius \( r < R \). A point on the wheel a distance \( d \) from its center traces out a curve called an epicycloid. (The Spirograph curves are examples of these.) The parametrization of these curves is

\[
\begin{align*}
x &= (R - r) \cos(t) + d \cos((R - r)t/r) \\
y &= (R - r) \sin(t) - d \sin((R - r)t/r)
\end{align*}
\]

For each of the following choices of \( R \), \( r \) and \( d \), plot the curve. Plotting over \( 0 \leq t \leq 2\pi \) may not give the entire curve. In each case, determine the smallest range of \( t \) that produces the whole curve (each is some integer multiple of \( \pi \)). How many loops does each curve have?

(a) \( R = 96, \ r = 64, \ d = 30 \).
(b) \( R = 96, \ r = 72, \ d = 20 \).
(c) \( R = 96, \ r = 56, \ d = 48 \).
(d) \( R = 96, \ r = 74, \ d = 60 \).
(e) \( R = 96, \ r = 52, \ d = 40 \).