Math 375: Probability Theory

Spring 2018

Homework 8

- Exercises from the text: 5.92, 5.94, 5.103, 5.106, 5.108, 5.110, 5.114, 5.123, 5.124, 5.126
- Non-textbook problems.

  1. (a) Let Y be any random variable. Prove that $E(Y^2) \geq 0$, and that $E(Y^2) = 0$ if and only if $Y = 0$.
     (b) Prove that for any random variables X_1 and X_2, $E(X_1X_2)^2 \leq E(X_1^2)E(X_2^2)$. Hint: Consider $g(t) = E((X_2 + tX_1)^2)$.
     (c) Prove that if $X_2 = aX_1$ for some real number a, then $E(X_1X_2)^2 = E(X_1^2)E(X_2^2)$.
     (d) Conversely, prove that if $E(X_1X_2)^2 = E(X_1^2)E(X_2^2)$, and if $E(X_1^2) \neq 0$, then $X_2 = aX_1$, where $a = \frac{E(X_1X_2)}{E(X_1^2)}$. Hint: Show $E((X_2 - aX_1)^2) = 0$.
  2. Let Y be the trial on which the r^{th} success occurs in a binomial experiment with probability of success p. Recall that the distribution for Y is the negative binomial distribution.
     (a) Let X_1 be the trial on which the first success occurs, and let X_j be the number of trials *between* success $(j - 1)$ and success j, for $j > 1$. (More precisely, $X_j = Y_j - Y_{j-1}$, where Y_j is the trial on which success j occurs.) Express Y in terms of the X_j.
     (b) Explain why the X_j are independent random variables with geometric distribution. What property of the geometric distribution does this rely on?
     (c) Prove that $E(Y) = r/p$ and $V(Y) = r(1 - p)/p^2$.