Math 375: Probability Theory
Spring 2018
Homework 7

- Exercises from the text: 4.126, 4.130, 4.136, 4.141, 5.8, 5.14, 5.18, 5.26, 5.35, 5.48, 5.52, 5.58, 5.62, 5.64, 5.70, 5.76, 5.78, 5.81, 5.84
- Non-textbook problems.
  1. Suppose the radius R and height H of a cylinder are chosen independently and uniformly from the interval $[0, 2]$. Let V denote the volume of the cylinder. Find the expected values of R, H and V.
  2. Suppose Anthony is playing darts and aims for the bullseye, which is 1.25 inches in diameter. Also suppose that his throws are never further than 3 inches from the center. A possible model for the density of the location (Y_1, Y_2) of the throw is $f(y_1, y_2) = c(9 - y_1^2 - y_2^2)$ if $y_1^2 + y_2^2 \leq 9$ and $f(y_1, y_2) = 0$ otherwise.
     (a) Find c.
     (b) Find the probability Anthony hits the bullseye.
     (c) Find the expected value of the distance from the center.
  3. Suppose the random variables Y_1 and Y_2 are independent and that their joint density $f(y_1, y_2)$ depends only on the quantity $y_1^2 + y_2^2$ (such a function is called *radial* since it depends only on the distance from the origin). Show that both Y_1 and Y_2 are normally distributed, as follows.
     (a) Show that $\frac{f_1'(y_1)}{2y_1 f_1(y_1)} = \frac{f_2'(y_2)}{2y_2 f_2(y_2)}$.
     (b) Explain why both sides must equal some constant k and solve the differential equation
     $$\frac{f_1'(y_1)}{2y_1 f_1(y_1)} = k$$
     (c) Set $k = -1/2\sigma^2$ (why must k be negative?) and show that $f_1(y_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y_1^2}{2\sigma^2}}$.
  4. (Extra Credit) The following question is from the Putnam Competition: A dart, thrown at random, hits a square target. Assuming that any point on the target area is equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge. Express your answer in the form $(a\sqrt{b} + c)/d$, where a, b, c and d are integers.