Math 375: Probability Theory
Spring 2018
Homework 2

- Exercises from the text: 2.71, 2.73, 2.75, 2.76, 2.94, 2.95, 2.97, 2.102, 2.104, 2.105, 2.110, 2.114, 2.121, 2.124, 2.128, 2.133
- Non-textbook problems.

  1. Suppose A and B are independent. Show that each of the following pairs are also independent: A and $\overline{B}$, $\overline{A}$ and B, $\overline{A}$ and $\overline{B}$.
  2. Suppose A and B are mutually exclusive events with $0 < P(A) < 1$ and $0 < P(B) < 1$. Can A and B be independent? Prove your assertion.
  3. Suppose A and B are events with $0 < P(A) < 1$ and $0 < P(B) < 1$ such that $A \subset B$. Can A and B be independent? Prove your assertion.
  4. Express $P(A \cup B \cup C \cup D)$ in terms of the probabilities of A, B, C, D, and intersections involving these sets.
  5. Three events A, B and C are **pairwise independent** if $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$ and $P(B \cap C) = P(B)P(C)$. That is, any two of the three events are independent. The three events are **mutually independent** if they are pairwise independent and $P(A \cap B \cap C) = P(A)P(B)P(C)$.
     (a) Consider rolling a fair 4-sided die numbered 1 through 4, and define $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{3, 1\}$. Prove that A, B, and C are pairwise independent, but not mutually independent.
     (b) Prove that if A, B, and C are mutually independent, then $P(A|B \cap C) = P(A)$, $P(B|A \cap C) = P(B)$ and $P(C|A \cap B) = P(C)$.
     (c) Consider rolling a 6-sided die that is "loaded" so that the probabilities of rolling 1, 2 or 3 are each 8/27 and the probabilities of rolling 4, 5, or 6 are each 1/27. Let $A = \{1, 4\}$, $B = \{2, 4\}$ and $C = \{3, 4\}$. Prove that $P(A \cap B \cap C) = P(A)P(B)P(C)$, but that A, B, and C are not pairwise independent.