**College of the Holy Cross, Fall 2018**
**Math 244, Homework 9**

1. For each matrix do the following:
   - Find all real eigenvalues.
   - For each eigenvalue $\lambda$, find a basis for $E_\lambda$.
   - Determine whether or not the matrix is diagonalizable, and if it is, find an eigenbasis.

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   (a) $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & 4 \\ 9 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 3 & 0 \\ 5 & 3 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

   (e) $A = \begin{bmatrix} 2 & 0 & 0 \\ 5 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$

   (f) $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$

2. Suppose $A$ and $B$ are similar matrices, so $B = C^{-1}AC$ for some matrix $C$.
   (a) Show that the characteristic polynomials of $A$ and $B$ are the same, and thus $A$ and $B$ have the same eigenvalues.
   (b) Show that if $\mathbf{v}$ is an eigenvector of $B$ with eigenvalue $\lambda$, then $C\mathbf{v}$ is an eigenvector of $A$ with eigenvalue $\lambda$.

3. Let $A$ be an invertible matrix.
   (a) Show that 0 is not an eigenvalue of $A$.
   (b) Suppose $\mathbf{v}$ is an eigenvector of $A$ with eigenvalue $\lambda$. Show that $\mathbf{v}$ is an eigenvector of $A^{-1}$ with eigenvalue $\lambda^{-1}$.
   (c) Show that $A$ is diagonalizable if and only if $A^{-1}$ is diagonalizable.

4. Let $W$ be a subspace of $\mathbf{R}^n$.
   (a) Prove that $W^\perp$ is a also subspace of $\mathbf{R}^n$.
   (b) Prove that $W \cap W^\perp = \{\mathbf{0}\}$
   (c) Prove $(W^\perp)^\perp = W$.

5. Let $W_1$ and $W_2$ be subspaces of $\mathbf{R}^n$. Prove that $(W_1 + W_2)^\perp = W_1^\perp \cap W_2^\perp$.

6. Use the Gram-Schmidt process to find an orthonormal basis for the plane in $\mathbf{R}^3$ spanned by $(1, 2, 3)$ and $(2, 0, -1)$.

7. Use the Gram-Schmidt process to find an orthonormal basis for the subspace of $\mathbf{R}^4$ spanned by $(1, 1, 1, 1)$, $(0, 0, 1, 1)$ and $(1, 0, 1, 0)$.