College of the Holy Cross, Fall 2018
Math 244, Homework 8

1. Find the determinant of each matrix.

   (a) $\begin{bmatrix} 3 & 7 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 11 & 0 \\ 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 11 \\ 4 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 11 \\ 0 & 4 \end{bmatrix}$

2. Find the determinant of the matrix for each transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ with respect to the standard basis.

   (a) Rot_θ (b) Ref_θ (c) $\mathrm{Proj}_{\mathbf{a}}$

3. Give an example of 2×2 matrices A and B such that $\det(A + B) \neq \det(A) + \det(B)$.

4. Show that for any 2×2 matrix A and any $c \in \mathbf{R}$, $\det(cA) = c^2 \det(A)$.

5. Let A and B be 2×2 matrices. Prove that $\det(AB) = \det(A)\det(B)$.

6. Suppose A is a 2×2 invertible matrix. Show that $\det(A^{-1}) = \dfrac{1}{\det(A)}$.

7. Find the determinant of each matrix.

   (a) $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 1 & 1 \\ -3 & 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 4 & 7 \\ 5 & 7 & 9 \end{bmatrix}$ (f) $\begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 7 & 2 & 0 & 0 & 0 \\ -3 & 6 & -1 & 0 & 0 \\ 4 & -8 & 3 & 7 & 0 \\ 9 & 6 & 3 & 7 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 7 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & 7 & 2 \\ 0 & 5 & 9 \\ 0 & 0 & -3 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 5 & 3 & 2 \\ 1 & 2 & 4 & 4 \\ 2 & 1 & 2 & 1 \\ 2 & -1 & 4 & 3 \end{bmatrix}$ (g) $\begin{bmatrix} 2 & 1 & 3 & 1 & 2 \\ 6 & 2 & 11 & 5 & 3 \\ 4 & -8 & 3 & 7 & 13 \\ 2 & 1 & 3 & 1 & 2 \\ 9 & 6 & 3 & 7 & 3 \end{bmatrix}$

8. For which real numbers x is each of the following matrices invertible?

   (a) $\begin{bmatrix} 1-x & 2 \\ 3 & 5-x \end{bmatrix}$ (b) $\begin{bmatrix} 1 & x \\ x & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix}$ (d) $\begin{bmatrix} x & 1 & 2 \\ 1 & x & 1 \\ 2 & 1 & x \end{bmatrix}$

9. The transpose of a matrix A is the matrix A^t whose entry in row i column j is the entry of A in row j column i. For example, if

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad \text{then} \qquad A^t = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}.$$

   Prove that for any 3×3 matrix A, $\det(A^t) = \det(A)$.