College of the Holy Cross, Spring Semester, 2019
Math 134 Worksheet 17
Due Wednesday, April 24

1. Suppose we want to express the function $f(x) = \sin(x) + \cos(x)$ as a power series

$$\sin(x) + \cos(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots$$

   (a) Plug in $x = 0$ to this formula. What must a_0 be?

   (b) Now take the derivative of both sides and plug in $x = 0$. What must a_1 be?

   (c) Take the derivative of both sides again and plug in $x = 0$. What must a_2 be?

   (d) What must a_3 be?

   (e) Compute a_4 through a_8.

   (f) Use Desmos to plot the partial sums s_n for $n = 1$ through $n = 8$, together with the function $f(x)$ on the domain $[-\pi, \pi]$.

2. Suppose we want to express $g(x) = \sqrt{1+x}$ as a power series

$$\sqrt{1+x} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + \cdots$$

   (a) Use the same method as in question 1 to find b_0 through b_5.

   (b) Plot the partial sums s_n for $n = 1$ through $n = 5$, together with the function $g(x)$ on the domain $[-1, 1]$.