College of the Holy Cross, Spring Semester, 2019
Math 134 Worksheet 16
Due Wednesday, April 17

1. Determine whether the given information implies that the series $\sum_{n=1}^{\infty} a_n$ (A) must converge, (B) must diverge, or (C) could converge or diverge. If you answer (A) or (B), state which test applies, and if you answer (C) give an examples of two series satisfying the given condition, one of which converges and one of which diverges.

   (a) $\lim_{n\to\infty} a_n = \frac{2}{5}$

   (b) $\lim_{n\to\infty} a_n = 0$

   (c) $0 \leq a_n \leq \frac{1}{n^2}$ for all n

   (d) $a_n \geq \frac{1}{n^2}$ for all n

   (e) $0 \leq a_n \leq \frac{1}{\sqrt{n}}$ for all n

   (f) $a_n \geq \frac{1}{\sqrt{n}}$ for all n

   (g) $\lim_{n\to\infty} \frac{a_n}{\frac{1}{n^2}} = 5$

   (h) $\lim_{n\to\infty} \frac{a_n}{\frac{1}{n}} = \frac{1}{3}$

   (i) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 5$

   (j) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{4}{5}$

   (k) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$

   (l) $\sum_{n=1}^{\infty} |a_n|$ converges

   (m) $\sum_{n=1}^{\infty} |a_n|$ diverges

   (n) $a_n = f(n)$ and $\int_1^{\infty} f(x)\,dx = 12$

2. Use the comparison test or limit comparison test to determine whether each the series converges or diverges.

   (a) $\sum_{n=1}^{\infty} \frac{1}{(n^2+4)^2}$

   (b) $\sum_{n=1}^{\infty} \frac{3n^3+1}{n^4+7n^2+6}$

   (c) $\sum_{n=1}^{\infty} \frac{3^n+4^n}{2^n+5^n}$

3. Use the alternating series error bound to determine how large N must be in order for S_N to approximate the the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4+1}$ to within 0.0001. Compute S_N for this N.

4. Use the ratio test to determine whether each series converges or diverges.

   (a) $\sum_{n=1}^{\infty} \frac{3^n}{n^4}$

   (b) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

   (c) $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$