

College of the Holy Cross, Spring Semester, 2019  
Math 134 Worksheet 15  
Due Tuesday, April 9

1. Evaluate each of the following.

(a)  $\sum_{n=1}^{\infty} \frac{e^{n+1}}{\pi^{n-1}}$ .

(b)  $\sum_{n=0}^{20} 5(4^n)$

(c)  $\sum_{n=1}^{\infty} \frac{2(3^n) + 7^n}{10^n}$

(d)  $\sum_{n=1}^{\infty} \left(\frac{\pi}{3}\right)^n$

2. At dinner one night, David helps himself a one cup serving of soup. After finishing this, he decides to have more, but is not as hungry, so only takes  $\frac{3}{4}$  of a cup of soup. After finishing this, he takes a third helping of  $\frac{9}{16}$  of a cup of soup, then a fourth helping of  $\frac{27}{64}$  of a cup. Assuming this pattern continues indefinitely, how many cups of soup does David eat altogether?

3. Suppose you take out a loan of some amount  $P$  at an annual interest rate  $r$  compounded monthly, and that you make payments of  $A$  dollars per month. This means that at the end of each month we take the current balance and multiply it by  $s = 1 + \frac{r}{12}$  (to add the interest to the balance) and then subtract  $A$  (to deduct the payment amount from the balance). So at the end of 1 month the new balance on the loan will be  $B_1 = sP - A$ , and after 2 months the balance will be

$$B_2 = sB_1 - A = s(sP - A) - A = s^2P - sA - A$$

(a) What is the balance  $B_3$  after 3 months? What is the balance  $B_4$  after 4 months? What is the balance  $B_m$  after  $m$  months? Use the formula for a finite geometric series to simplify this.

(b) Suppose you want to pay off the loan in  $m$  months. What should the monthly payment  $A$  be in terms of  $P$ ,  $r$  and  $m$ ? (Hint: Set  $B_m = 0$  and solve for  $A$ .)

(c) Apply your formula to find the monthly payment on a 10 year loan of \$10,000 with interest rate 5%. How much money do you actually wind up paying during the 10 year period?

4. Use the integral test to determine whether each series converges or diverges.

(a)  $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

(b)  $\sum_{n=1}^{\infty} \frac{n}{n^2 + 8}$