College of the Holy Cross, Spring Semester, 2019
Math 134 Worksheet 13
Due Friday, March 29

1. (a) Write the formulas for the trapezoid rule and Simpson's rule approximations of $\int_a^b f(x)\,dx$.

   $T_N =$

   $S_N =$

   (b) Explain how each term in these formulas is computed.

2. Consider the integral $\int_1^4 \sqrt{x}\,dx$.

   (a) Compute the approximations T_6 and S_6 of this integral.

   (b) Compute the exact value of the integral and use it to find the error in each approximation. Which approximation is better?

3. (a) Write the formulas for the error bounds for the trapezoid rule and Simpson's rule approximations of $\int_a^b f(x)\,dx$.

   $E(T_N) \leq$ $\qquad\qquad$ $E(S_N) \leq$

   (b) Explain how the terms K_2 and K_4 are determined.

   (c) Explain what these formulas tell us.

   (d) Give an example of a non-constant function $f(x)$ for which $K_2 = 0$. (In this case the trapezoid rule approximation would have zero error.)

   (e) Give an example of a function $f(x)$ for which $K_2 \neq 0$ but $K_4 = 0$. (In this case the Simpson's rule approximation would have zero error.)

   (f) In the Simpson's rule error bound, what is the effect of multiplying N by 10?

4. Consider the definite integral $\int_0^2 \sin(x^2)\,dx$.

   (a) Compute T_4 and S_4.

   (b) Find the second derivative of $f(x) = \sin(x^2)$ and use a calculator or computer to determine an upper bound K_2 for $|f''(x)|$ on the interval $[0, 2]$.

   (c) Find an upper bound for $E(T_4)$.

   (d) How large must N be in order for T_N to approximate the integral to within 10^{-6}?

   (e) The fourth derivative of $f(x) = \sin(x^2)$ is $f''''(x) = 4(4x^4 - 3)\sin(x^2) - 48x^2\cos(x^2)$. Use a calculator or computer to determine an upper bound K_4 for $|f''''(x)|$ on the interval $[0, 2]$.

   (f) Find an upper bound for $E(S_4)$.

   (g) How large must N be in order for S_N to approximate the integral to within 10^{-6}?

   (h) Use a calculator or computer to calculate S_N for the value of N found in part (d).