College of the Holy Cross, Spring Semester, 2019
Math 134 Worksheet 12
Due Tuesday, March 26

Recall that if a random variable X has probability density function $p(x)$, then

$$P(a \leq X \leq b) = \int_a^b p(x)\,dx \qquad \text{and} \qquad \mu = \int_{-\infty}^{\infty} xp(x)\,dx$$

The **median** of X is the value m such that $P(X \leq m) = P(x \geq m) = \frac{1}{2}$. Thus m is the number such that

$$\int_{-\infty}^{m} p(x)\,dx = \int_m^{\infty} p(x)\,dx = \frac{1}{2}$$

The **variance** of X is

$$\text{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\,dx$$

and the **standard deviation** of X is $\sigma = \sqrt{\text{Var}(X)}$.

1. Suppose a random variable X has probability density function $p(x) = k(4-x)$ for $0 \leq x \leq 4$ and $p(x) = 0$ for all other x.
   (a) Find k and sketch the function $p(x)$.
   (b) Find $P(1 \leq X \leq 3)$.
   (c) Find $P(X \geq 3)$.
   (d) Find the mean of X.
   (e) Find the median of X.
   (f) Find the variance and standard deviation of X.

2. Let X be the time it takes for a customer to check out at a particular store. Suppose X is modelled by an exponential density function with mean 4 minutes.
   (a) Write the formula for the density $p(x)$.
   (b) Find the probability that a customer takes between 3 and 5 minutes to check out.
   (c) Find the probability that a customer takes more than 6 minutes to check out.
   (d) Find the median check out time.
   (e) Find the 80th percentile of checkout times. That is, find the time t such that 80 percent of customers have check out time less than or equal to t.

3. According to marathonguide.com, the average finishing time in marathons in the U.S. in 2010 was about 4 hours and 35 minutes, with a standard deviation of 1 hour and 2 minutes. For this exercise, assume marathon times are normally distributed.
   (a) Find the probability that a randomly selected marathon runner in 2010:
      (i) took between 4 and 5 hours to run a marathon.
      (ii) took over 5 hours.
      (iii) took under 3 hours.
   (b) What time (in hours and minutes) would someone need to run in order to be in the top 10% (fastest) of marathon times?