College of the Holy Cross, Spring Semester, 2019
Math 134: Calculus 2 with Fundamentals

Trigonometric Integrals

To evaluate integrals of the form

$$\int \sin^n(x)\,dx \qquad \int \cos^n(x)\,dx \qquad \int \sec^n(x)\,dx \qquad \int \tan^n(x)\,dx$$

use the appropriate reduction formula, formulas 44, 45, 46, and 48 in the back of the book (these will be provided on quizzes and exams if needed.) You should memorize these integrals for the case $n = 1$, and the case $n = 2$ for secant and tangent. That is, you should know formulas 5, 6, 11, 13, 7 and 36.

To evaluate integrals of the form

$$\int \sin^m(x)\cos^n(x)\,dx$$

do the following.

- If n is an odd positive integer, substitute $u = \sin(x)$.
- If m is an odd positive integer, substitute $u = \cos(x)$.
- If both m and n are positive even integers, use the identity $\sin^2(x) + \cos^2(x) = 1$ to rewrite the integrand in terms of just $\sin(x)$ or just $\cos(x)$ and use reduction.

To evaluate integrals of the form

$$\int \sec^m(x)\tan^n(x)\,dx$$

do the following.

- If m is an even positive integer, substitute $u = \tan(x)$.
- If n is an odd positive integer, substitute $u = \sec(x)$.
- If m is an odd positive integer and n is an even positive integer, use the identity $\tan^2(x) + 1 = \sec^2(x)$ to rewrite the integrand in terms of just $\sec(x)$ or just $\tan(x)$ and use reduction.

Trigonometric Substitution

To evaluate integrals involving the expressions $a^2 - x^2$, $a^2 + x^2$ and $x^2 - a^2$, do the following.

- $a^2 - x^2$: Let $x = a\sin(u)$. Then $dx = a\cos(u)\,du$ and $a^2 - x^2 = a^2\cos^2(u)$.
- $a^2 + x^2$: Let $x = a\tan(u)$. Then $dx = a\sec^2(u)\,du$ and $a^2 + x^2 = a^2\sec^2(u)$.
- $x^2 - a^2$: Let $x = a\sec(u)$. Then $dx = a\sec(u)\tan(u)\,du$ and $x^2 - a^2 = a^2\tan^2(u)$.