Math 134: Calculus 2 with Fundamentals
Spring 2019
Professor Levandosky
Practice Integrals

1. $\displaystyle\int \frac{x^3}{x^4-1}\,dx = \frac{1}{4}\ln|x^4-1| + C$ (substitute $u = x^4 - 1$.)

2. $\displaystyle\int \frac{x^2}{x^4-1}\,dx = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2}\tan^{-1}(x) + C$

   Use partial fractions: $\displaystyle\frac{x^2}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$

   Then $x^2 = A(x^2+1)(x+1) + B(x^2+1)(x-1) + (Cx+D)(x-1)(x+1)$.

   Setting $x=1$ gives $A = \frac{1}{4}$, setting $x=-1$ gives $B = -\frac{1}{4}$, setting $x = 0$ gives $D = \frac{1}{2}$, and setting $x = 2$ gives $C = 0$.

3. $\displaystyle\int \frac{x}{x^4+1}\,dx = \frac{1}{2}\tan^{-1}(x^2) + C$ (Write $x^4 = (x^2)^2$ and substitute $u = x^2$)

4. $\displaystyle\int x^3\tan^{-1}(x)\,dx = \frac{1}{4}x^4\tan^{-1}(x) - \frac{1}{4}\left(\frac{1}{3}x^3 - x + \tan^{-1}(x)\right) + C$

   First use parts with $u = \tan^{-1}(x)$ and $dv = x^3\,dx$ to get $\displaystyle\frac{1}{4}x^4\tan^{-1}(x) - \frac{1}{4}\int \frac{x^4}{x^2+1}\,dx$

   Then use long division to write $\displaystyle\frac{x^4}{x^2+1} = x^2 - 1 + \frac{1}{x^2+1}$.

5. $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx = 2\left(\sqrt{x} - \ln|1+\sqrt{x}|\right) + C$ (substitute $u = 1+\sqrt{x}$)

6. $\displaystyle\int e^{x^{1/3}}\,dx = e^{x^{1/3}}\left(3x^{2/3} - 6x^{1/3} + 6\right) + C$ (substitute $w = x^{1/3}$, then integrate by parts twice)

7. $\displaystyle\int \frac{1}{2+e^{-x}}\,dx = \frac{1}{2}\ln|2e^x+1| + C$ (multiply numerator and denominator by e^x, then substitute $u = 2e^x + 1$)

8. $\displaystyle\int x^3\ln(x)\,dx = \frac{1}{16}x^4\left(4\ln(x) - 1\right) + C$ (parts with $u = \ln(x)$ and $dv = x^3\,dx$)

9. $\displaystyle\int \sqrt{1+e^x}\,dx = 2\sqrt{1+e^x} + \ln|\sqrt{1+e^x} - 1| - \ln|\sqrt{1+e^x}+1| + C$

   First substitute $u = \sqrt{1+e^x}$ to get $\displaystyle\int \frac{2u^2}{u^2-1}\,du$.

   By long division this becomes $\displaystyle\int 2 + \frac{2}{u^2-1}\,du$. Then use partial fractions.

10. $\int \sqrt{x^2+x^4}\,dx = \frac{1}{3}(1+x^2)^{3/2} + C$ (Rewrite the integrand as $x\sqrt{1+x^2}$ and substitute $u = 1+x^2$)

11. $\int \frac{e^x}{e^{2x}+4}\,dx = \frac{1}{2}\tan^{-1}(\frac{1}{2}e^x) + C$ (Write $e^{2x} = (e^x)^2$ and substitute $u = e^x$)

12. $\int x\sqrt{4-x^4}\,dx = \sin^{-1}\left(\frac{x^2}{2}\right) + \frac{1}{4}x^2\sqrt{4-x^4} + C$

    First write $x^4 = (x^2)^2$ and substitute $w = x^2$ to get $\int \frac{1}{2}\sqrt{4-w^2}\,dw$.

    Then use the trigonometric substitution $w = 2\sin(u)$.

13. $\int \ln(x^2+1)\,dx = x\ln(x^2+1) - 2x + 2\tan^{-1}(x) + C$ (Integrate by parts with $u = \ln(x^2+1)$ and $dv = dx$, and the use long division.)

14. $\int \frac{\sqrt{x}}{x^3+1}\,dx = \frac{2}{3}\tan^{-1}(x^{3/2}) + C$ (substitute $u = x^{3/2}$)

15. $\int \frac{1}{\sqrt{4x^2-1}}\,dx = \frac{1}{2}\ln|2x + \sqrt{4x^2-1}| + C$ (trigonometric substitution, $x = \frac{1}{2}\sec(u)$)