Math 134, Spring 2019
Professor Levandosky
Practice Final Exam Questions

1. Suppose f is a continuous function on the interval $[1,4]$ with values given in the table below.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	3	6	10	16	21	27	30

   (a) Compute the Simpson's Rule approximation of $\int_1^4 f(x)\,dx$ that uses 6 subintervals.

   (b) Suppose $|f''''(x)| \le 10$ for all x in $[1,4]$. Find a bound on the error in the approximation in part (b).

2. Evaluate the following integrals.

   (a) $\displaystyle\int_2^\infty xe^{-x^2}\,dx$

   (b) $\displaystyle\int_0^{\pi/2} x\cos(3x)\,dx$

   (c) $\displaystyle\int \frac{2x}{(x-2)^2(x+4)}\,dx$

   (d) $\displaystyle\int \frac{(x^2-4)^{3/2}}{x}\,dx$

3. Let $P(t)$ denote the quantity (in gallons) of a chemical in a lake at time t (in weeks), and suppose that $P(0) = 350$ and $P'(t) = 200e^{-0.5t}$. Find $P(4)$.

4. Find the solution of $\dfrac{dy}{dx} = \dfrac{y^2}{x^2}$ that satisfies the condition $y(1) = 2$.

5. Let R be the region bounded by the curve $y = 1 + 2x$ and $y = (x-1)^2$.

   (a) Sketch R.

   (b) Find the area of R.

   (c) Find the volume of the solid obtained by revolving R about the x-axis.

6. Find the exact value of the sum of each series below.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-3)^n}{4^{n+1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty}(a_n + 3b_n)$, given that $\displaystyle\sum_{n=1}^{\infty} a_n = 100$ and $\displaystyle\sum_{n=1}^{\infty} b_n = 12$

   (c) $\displaystyle\sum_{n=1}^{\infty} c_n$, assuming $\displaystyle\sum_{n=1}^{N} c_n = \frac{50N}{2N+7}$ for all $N \ge 1$.

7. In each part below, consider the given information and decide whether the series $\sum_{n=1}^{\infty} a_n$

(A) must converge. Specify which convergence test applies.

(B) must diverge. Specify which convergence test applies.

(C) could converge or diverge. Provide <u>two</u> examples of series that satisfy the given conditions, one which converges and one which diverges.

Explain your reasoning.

(a) $\lim_{n\to\infty} a_n = 0$

(b) $\lim_{n\to\infty} a_n = \frac{1}{4}$

(c) $0 < a_n < \frac{7}{n}$ for all n

(d) $a_n > 0$ for all n and $\lim_{n\to\infty} \frac{a_n}{\frac{1}{\sqrt{n}}} = \frac{2}{3}$

8. Let $f(x) = (1+x)^{1/3}$.

(a) Find the degree 3 Taylor polynomial $T_3(x)$ for f centered at $x = 0$.

(b) Use the answer to part (a) to approximate $f(1) = \sqrt[3]{2}$. Round your answer to 3 decimal places.

(c) Find a bound on the error in the approximation in part (b).

9. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n^2 3^n}$. Be sure to test for convergence at the endpoints.

10. Let $P(t)$ denote the balance of an annuity that earns 3% interest compounded continuously and pays out \$24,000 per year continuously. As shown in class, this is modeled by the differential equation

$$\frac{dP}{dt} = 0.03P - 24000$$

(a) Find the equilibrium solution of the differential equation.

(b) Find the solution that satisfies $P(0) = 500000$.

(c) Find the time when the annuity runs out of money if $P(0) = 500000$.

11. Use the power series for e^x, $\sin(x)$ and $\cos(x)$ to do the following.

(a) Compute $\lim_{x\to 0} \frac{x\sin(x) - x^2}{\cos(x^2) - 1}$

(b) Express $\int_0^2 xe^{-x^3}\,dx$ as an infinite series.