Math 134, Spring 2019
Professor Levandosky
Practice Midterm Exam 3

1. Evaluate each of the following improper integrals.

   (a) $\displaystyle\int_0^\infty xe^{-7x}\,dx$ (b) $\displaystyle\int_1^5 \frac{1}{(x-1)^{3/2}}\,dx$

2. Let $p(x) = \dfrac{c}{x^3}$ for $x \geq 1$ and $p(x) = 0$ otherwise, and suppose p is the probability density function for a random variable X.

   (a) Find c.
   (b) Compute $P(2 \leq X \leq 3)$.
   (c) Find the mean of X.
   (d) Find the median of X.

3. In 2004 the mean math SAT score was 518 with a standard deviation of 114.

   (a) Find the probability that a randomly selected student scored between 600 and 700.
   (b) What score would be required to place in the top 10% of all math SAT scores that year?

4. (a) Use Simpson's Rule with $N = 6$ subintervals to approximate $\displaystyle\int_2^5 \ln(x)\,dx$. Write your answer in decimal form, to three decimal places.
   (b) Find a bound on the error in the approximation in part (a).

5. For each series, find its sum, or explain why it diverges.

   (a) $\displaystyle\sum_{n=1}^\infty \cos(1/n^2)$ (b) $\displaystyle\sum_{n=1}^\infty \frac{2^{2n-1}}{5^n}$ (c) $\displaystyle\sum_{n=1}^\infty 5^{1/n} - 5^{1/(n+1)}$

6. Consider the series $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n}{\sqrt{n}}$.

   (a) Find the first 4 partial sums of the series.
   (b) Show that the series converges.
   (c) How large must N be in order to ensure that the N^{th} partial sum to be within 10^{-6} of the sum of the series? (You don't need to compute this partial sum.)

7. (a) Use the integral test to determine whether the series $\displaystyle\sum_{n=1}^\infty \frac{1}{n(\ln(n)+4)^{1/2}}$ converges or diverges.
   (b) Determine whether the series $\displaystyle\sum_{n=1}^\infty \frac{3n^2}{2n^5+7}$ converges or diverges.
   (c) For which values of k does the series $\displaystyle\sum_{n=1}^\infty \frac{(k^2+1)^n}{n^3 2^n}$ converge?