College of the Holy Cross, Fall 2009
Math 351, Practice Midterm 2
Prof. Jones

1. Let H be a normal subgroup of a group G, and let K be any subgroup of G. Recall that in class we defined $HK \subset G$ to be

$$\{hk \mid h \in H, k \in K\}.$$

Prove that HK is a subgroup of G.

2. Let $|G| = p^n m$, where p is prime and $\gcd(p, m) = 1$. Suppose that H is a normal subgroup of G of order p^n. If K is a subgroup of G of order p^k for some $k \leq n$, show that $K \subseteq H$.

3. Let $G = \mathbb{Z}_4 \oplus \mathbb{Z}_4$, $H = \{(0,0), (2,0), (0,2), (2,2)\}$, and $K = \langle (1,2) \rangle$. Determine whether G/H is isomorphic to $\mathbb{Z}_4$ or $Z_2 \oplus \mathbb{Z}_2$. Do the same for G/K.

4. Show that $D_{11} \oplus \mathbb{Z}_3$ is not isomorphic to $D_3 \oplus \mathbb{Z}_{11}$.

5. Let $|a| = 20$. List the cosets of $\langle a^{15} \rangle$ in $\langle a \rangle$.

6. Show that in a group G of odd order, the equation $x^2 = a$ has a unique solution for all $a \in G$.