College of the Holy Cross, Spring 2009
Math 244, Practice Midterm 1
Prof. Jones

1. For each of the following subsets W of a vector space V, determine if W is a subspace of V. In each case either prove that W is a subspace or give a concrete reason why it is not a subspace.

   (b) $V = \mathbb{R}^4$, and $W = \{(x_1, x_2, x_3, x_4) \,|\, x_1 = x_3 - x_2, \text{ and } x_4 = x_1 x_3\}$

   (c) $V = M_{2\times 2}(\mathbb{R})$, and $W = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \,|\, a_{21} = 0 \right\}$

2. Given subspaces W_1 and W_2 of a vector space V, recall that
$$W_1 + W_2 = \{\mathbf{x} \in V \,|\, \mathbf{x} = \mathbf{w_1} + \mathbf{w_2} \text{for some } \mathbf{w_1} \in W_1 \text{ and } \mathbf{w_2} \in W_2\}.$$
   Prove that $W_1 + W_2$ is a subspace of V.

3. Let $V = \mathbb{R}^2$ and $S = \{(2, 2), (2, -2)\}$.

   (a) Show that $(1, 2)$ is in $\text{Span}(S)$.

   (b) Show that every vector (b_1, b_2) in $\mathbb{R}^2$ is in $\text{Span}(S)$.

4. Let V be a vector space, and S a non-empty subset of V. Assume that each vector in $\text{Span}(S)$ can be written in one and only one way as a linear combination of vectors in S. Show that S is linearly independent.

5. Determine whether each of the following sets of vectors is linearly independent of linearly dependent. In each case prove your assertion.

   (a) $S = \{1, x, x^2\}$ in $P_2(\mathbb{R})$.

   (b) $S = \{(0, 2, 4), (0, 1, 3), (1, 1, 4)\}$ in $\mathbb{R}^3$.

   (c) $S = \{e^x, e^{2x}, \sqrt{x}\}$ in $F(\mathbb{R})$.

6. Find a parametrization of the set of solutions of the following system of linear equations.
$$\begin{array}{rrrrl} x_1 & +2x_2 & & +x_4 & = 0 \\ -3x_1 & & +x_3 & & = 1 \\ -x_1 & +4x_2 & +x_3 & +2x_4 & = 1 \end{array}$$