College of the Holy Cross, Fall 2008
Math 243, Practice Midterm 3

The problems on the actual exam will be somewhat similar to the problems here but some problems could look very different (in form or degree of difficulty) from the problems here. This practice exam is a bit longer than the actual exam will be. Working on this practice exam should **NOT** be your only preparation for the exam. Make sure you understand how to do all the problems on the homework. The third exam covers sections 2.3, 2.4, 2.5, and 2.6.

1. (a) Give the definition of the term: an integer a *divides* an integer b.
   (b) Suppose that a, b and c are integers such that $a|b$ and $b|c$. Prove that $a|c$.

2. Use induction to prove that 3 divides $n^3 + 2n$ for all integers $n \geq 1$.

3. Use the Euclidian algorithm to find the greatest common divisor of 5088 and 156.

4. Given the congruence $9x + 19 \equiv 2 \pmod{23}$, find a solution x such that $0 \leq x < 23$.

5. List all the elements of $\mathbb{Z}_{30}$ that have multiplicative inverses. Find the inverse of [23].

6. Prove that the equation $x^2 \equiv 2 \pmod 3$ has no solutions $x \in \mathbb{Z}$. (Hint: Use the Division Algorithm and consider the possible remainders when x is written $3q + r$.)

7. Let $d = (a, n)$. Prove that if the congruence $ax \equiv b \pmod n$ has a solution, then $d \mid b$. (Note: this shows, for instance, that $8x \equiv 6 \bmod 12$ has no solution, which was something you found on the past HW assignment.)

8. Suppose p is a prime. Prove that [1] and $[p - 1]$ are the only elements in $\mathbb{Z}_p$ that are their own multiplicative inverses. (Hint: Assume an element of $\mathbb{Z}_p$ is its own inverse and show that it has to be [1] or $[p - 1]$.)