

College of the Holy Cross, Fall 2008
Math 243, Practice Midterm 1

1. Negate the following statement: *There is a politician who is either honest or not trustworthy.* Do **not** use “There is no politician...”
2. Write the converse, inverse and contrapositive of the following implication, and determine the truth or falsity of each statement (including the implication). You do **not** need to prove the truth or falsity of the statement.
Implication: *If x and y are integers divisible by 3, then $x + y$ is an integer divisible by 3.*

3. Let A and B be sets. Prove that if $A \subseteq B$, then $A \cap C \subseteq B \cap C$.

4. Let $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ be given by

$$f(x, y) = (x + y, 1).$$

Is f onto? Is f one-to-one? In each case prove your conclusion.

5. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and let $\mathcal{P}(A)$ be the power set of A . Consider the map $f : A \rightarrow \mathcal{P}(A)$ that sends each $x \in A$ to the set $A - \{x\}$. Thus

$$f(1) = \{2, 3, 4, 5, 6, 7, 8, 9\}.$$

Is f onto? Is f one-to-one? In each case prove your conclusion.

6. Prove or disprove the following statement: for any sets A, B , and C , $A \cup B = A \cup C$ implies that $B = C$.

7. Compute $\mathcal{P}(\mathcal{P}(\emptyset))$, where \emptyset denotes the empty set.