Algebra problems, due Tuesday 9/9

These do not count as part of your grade, but you will get feedback on them.

1. What is the slope of the straight line through the points $P = (3, 5)$ and $Q = (6, 9)$? What is the equation of this line?

   The slope of this line is $\frac{9-5}{6-3}$, which is $4/3$, and we can use point-slope form with the point P to get
   $$y - 5 = \frac{4}{3}(x - 3),$$
   which is the same as $y = \frac{4}{3}x - 4 + 5$, which is $y = \frac{4}{3}x + 1$.

2. Solve the equation $x^2 + 3x - 7 = 0$.

   Using the quadratic formula, the solutions are
   $$x = \frac{-3 \pm \sqrt{3^2 - (4 * -7)}}{2} = \frac{-3 \pm \sqrt{37}}{2}.$$

3. Solve the inequality $x^2 - 5x + 6 > 0$.

   Factoring the left-hand side gives $(x - 3)(x - 2) > 0$. To solve this, one can make a sign diagram, or just note that it is true when both $(x - 3) < 0$ and $(x - 2) < 0$ or when both $(x - 3) > 0$ and $(x - 2) > 0$. The former happens when $x < 2$, and the latter when $x > 3$. So the solutions are all x with $x < 2$ or $x > 3$.

4. Solve the equation $P(V - b) = nRT$ for V.

   Dividing both sides by P gives
   $$V - b = \frac{nRT}{P}$$
   and then adding b to both sides yields
   $$V = \frac{nRT}{P} + b.$$

5. Write $(x + y)^3$ as a polynomial in x and y (in other words, a sum of terms like x^2y, and y^3).

   We have $(x + y)^3 = (x + y)(x + y)(x + y) = (x^2 + 2xy + y^2)(x + y)$. To do the last multiplication, we multiply the left-hand term by x, giving
   $$x^3 + 2x^2y + xy^2,$$
   and then multiply the left-hand term by y, giving
   $$x^2y + 2xy^2 + y^3,$$
   and then add these together, getting $x^3 + 3x^2y + 3xy^2 + y^3$.

6. Simplify the expression $a^3(3a-b)^2 - b^2(a^3+7b)$.

   We get, successively,

$$\begin{aligned} a^3(9a^2-6ab+b^2) - b^2(a^3+7b) \\ 9a^5 - 6a^4b + a^3b^2 - b^2a^3 - 7b^3 \\ 9a^5 - 6a^4b - 7b^3 \end{aligned}$$

7. Simplify the exression $\dfrac{3x+2}{x^2-x} - \dfrac{x+1}{x^2}$.

   Factoring the denominator of the left fraction, we get $x(x-1)$. So the common denominator we need is $x^2(x-1)$. We multiply the left fraction by x in numerator and denominator, and the right fraction by $x-1$ in numerator and denominator, giving

$$\frac{3x^2+2x}{x^2(x-1)} - \frac{(x+1)(x-1)}{x^2(x-1)},$$

   which is

$$\frac{3x^2+2x}{x^2(x-1)} - \frac{x^2-1}{x^2(x-1)},$$

   and this is

$$\frac{2x^2+2x+1}{x^2(x-1).}$$

8. Without a calculator, find the numerical value of $27^{2/3}$.

   This is the same as $(\sqrt[3]{27})^2$, which is 3^2, or 9.

9. Simplify the expression $\dfrac{y^4(x^3y^{-2})^2}{x^{-1}}$.

   Applying the square in the numerator gives $y^4x^6y^{-4}$, which is x^6. This leaves us with

$$\frac{x^6}{x^{-1}}.$$

   Now $1/x^{-1}$ is x^1, so x^6/x^{-1} is just x^6x, which is x^7.