MATH 392 – Seminar in Computational Commutative Algebra
Problem Set 3
**Due:** February 15, 2019

1. In the ring $\mathbb{Q}[x]$,

(a) Use the *Euclidean algorithm* for polynomials to compute the greatest common divisor of $f_1(x) = x^3 + 3x^2 - 2$ and $f_2(x) = x^4 - 1$.

(b) Find polynomials $A(x)$ and $B(x)$ such that

$$\gcd(f_1(x), f_2(x)) = A(x) \cdot f_1(x) + B(x) \cdot f_2(x).$$

(c) Is $f(x) = x^6 + 7x + 1$ in the ideal $\langle f_1(x), f_2(x)\rangle$?

2. In class we mentioned that there is a big analogy between the algebra of the ring of integers $\mathbb{Z}$ and algebra of the polynomial ring $k[x]$ in one variable over a field $k$ because we have similar division algorithms in both. What should the analog of a *prime number* $p \in \mathbb{Z}$ be in the polynomial ring $\mathbb{Q}[x]$? Try to give a precise definition. One thing to be careful of: Nonzero constants $m/n \in \mathbb{Q}$ are also elements of $\mathbb{Q}[x]$ but they have multiplicative inverses: $(m/n) \cdot (n/m) = 1$. This says you can always factor *any* nonzero constant out of the coefficients of a polynomial, for instance:

$$3x^3 + 4x + 1 = 3 \cdot \left(x^3 + \frac{4}{3}x + \frac{1}{3}\right) = \frac{1}{7} \cdot \left(21x^3 + 28x + 7\right),$$

etc. But those kinds of factorization are not the "interesting" ones. An "interesting" factorization would be something like

$$x^2 - 1 = (x - 1)(x + 1).$$

From the text: Chapter 2, section 2/2, 7, 10, 11; Chapter 2, section 3/1, 9